DOCUMENT RESUME

ED 277 583 SE 047 685

TITLE North Dakota Mathematics Curriculum Guide. Secondary (9-12).
INSTITUTION North Dakota State Dept. of Public Instruction, Bismarck.
PUB DATE Mar 86
NOTE 140p.; Document contains occasional small print.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC06 Plus Postage.
DESCRIPTORS Algebra; Computer Uses in Education; Course Content; *Course Objectives; Geometry; Mathematical Applications; *Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; *Mathematics Skills; Secondary Education; *Secondary School Mathematics; State Curriculum Guides; Statistics; Trigonometry
IDENTIFIERS *North Dakota

ABSTRACT

Developed to provide assistance and support to local schools that want to improve their secondary mathematics programs, this guide provides an updated view of what North Dakota educators feel is necessary in a secondary mathematics curriculum. An overview of perceived statewide mathematics objectives and a review of national perspectives on mathematics education are presented. A section explaining the role of the classroom teacher in dealing with special needs students is also included. The guide identifies seven core courses and describes each with a course outline, course objectives, and examples or comments on the topics. Courses reviewed include: (1) algebra; (2) geometry; (3) algebra II; (4) trigonometry; (5) analytic geometry; (6) probability and statistics; and (7) mathematical applications. Appendices consist of sections addressing the use of calculators, computers and textbook evaluation in mathematics education. Also included are a position paper on basic skills, an agenda for minimum skills for college algebra, and a bibliography. (ML)

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THE STATE OF NORTH DAKOTA

Department of Public Instruction

Dr. Wayne G. Sanstead, Superintendent

BISMARCK, NORTH DAKOTA 58505

MATHEMATICS CURRICULUM GUIDE

9-12

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March, 1986

NORTH DAKOTA
MATHEMATICS CURRICULUM GUIDE

SECONDARY (9-12)

Dr. Wayne G. Sanstead, SUPERINTENDENT
North Dakota Department of Public Instruction
Bismarck, North Dakota 58505

March, 1986

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FOREWORD

The North Dakota Secondary Mathematics Guide was developed to provide assistance and support to local schools that want to improve their secondary mathematics program. This Guide is not a mandate to local systems, but rather an additional resource to be used by local schools in formulating their unique mathematics curricula.

Mathematics cannot be effectively taught with lecture and worksheets alone. Today's educators must be aware of learning styles and developmental stages of students and use this knowledge to accommodate each student through the use of differentiated instruction, manipulatives, activities and technology. The increased pace of changing technology has made it increasingly essential for students to fully develop their mathematical abilities. Not only must a person be able to perform basic computational skills, but must be able to apply these skills to new applications.

In a rapidly changing society, people find it necessary to make practical solutions to problems that were not realized previously. Emphasis can no longer be placed merely on computational skills, but must be placed on problem solving skills as well. With the continued decreasing cost of minicalculators it is no longer necessary to burden people with time consuming computations. Once basic computation competency has been achieved, the integration of the calculator into the mathematics program is essential to allow students to focus additional attention on application problems.

In addition, students must be exposed to computer technology. This should not be an isolated experience, but integrated into the mathematics program as the computer has been integrated into society. The use of the computer as both a remedial and enrichment tool is merely a beginning. Educators must stay abreast of new uses for this valuable instrument.

Wayne G. Sanstead

Dr. Wayne G. Sanstead
Superintendent of Public Instruction

SMC: 3/86

ACKNOWLEDGEMENTS

In 1983, under the direction of Dr. Glenn Prigge, a state mathematics committee, consisting of educators from the state's colleges, universities, elementary, and high schools, was formed. Through the efforts of these people, the K-12 Mathematics Curriculum Guide was created.

A special thanks to the members of the Mathematics Curriculum Committee who have contributed so much of their time and efforts in making this Guide the useful document that it is.

K-12 Mathematics Curriculum Committee

Dr. Glenn Prigge, Director
University of North Dakota

University and College

Dr. Robert Holmen
Minot State College

Dr. Merwin Lyng
Mayville State College

Bill Massey
Dickinson State College

Dr. Ronald Mathsen
North Dakota State University

Roger Rand
Valley City State College

High School

Gary Froelich
Bismarck High School

Joan McCarthy
Shanley High School
Fargo

Bruce Moen
Hillsboro High School

Neil Pederson
Carrington High School

Judy Wolsky
Sargent Central
Forman

Consultants

Dr. Harold Bergquist
Dr. Lila Prigge
Dr. Ken Vos

Elementary

Maureen Koppelsloen
Central Elementary School
Tioga

Lenora Langdalen
Westhope/Wahpeton
Public Schools

Sally Moe
Beulah Middle School

Dolorisann Pletan
South Junior High
Grand Forks

Leona Wood
Jefferson Elementary
Minot

Thanks to the school administrators and instructors who took time to review the draft and submit their comments to the Department. Also, thanks to the North Dakota Curriculum Council for their involvement and help in this project.

A special note of appreciation is extended to the following Department staff for their efforts.

Dr. Wayne G. Sanstead, Superintendent
Elmer Huber, Deputy Superintendent
Ron Stastney, Assistant Superintendent, Instruction
Ron Torgeson, Information and Research
Pat Herbel, Director, Curriculum
Beth Fricke, Secretary
Jeanne Sandberg, Secretary
Paula Gabel, Secretary
Margo Undlin, Secretary

Charles DeRemer

Charles DeRemer
Assistant Director of Curriculum

NORTH DAKOTA MATHEMATICS
PHILOSOPHY AND GOALS

PHILOSOPHY*

There has been much discussion in regard to mathematics content and teaching approaches. The current approach is to incorporate traditional concepts with some modern materials. The traditional concepts are amplified and clarified. They are no longer presented as isolated, unrelated ideas or rules. This organized body of knowledge leads to logical thinking which gives a better understanding of the scientific world. The greatest challenge of teachers is to provide students with enough basic mathematics to meet their needs as well as to create within them the desire to pursue the study of mathematics beyond the minimum.

The mathematics curriculum must reflect the rapid changes in society (e.g. technology including the computer). Changes in mathematics instruction in the next decade probably will be motivated primarily by two things: an emphasis upon student learning and changes in technology. Teachers cannot be completely certain about the type of world in which students will be living or about the types of skills that will be required of them, but we do know that they will have to be able to adapt to new situations. Therefore, teachers must attempt to give them experience at facing new situations and working out solutions to problems that are new to them. There should be an emphasis on the discovery approach to learning as a way of continually confronting students with new situations. Furthermore, a rationale in terms of physical experiences or previously learned concepts should be given for each new concept that is learned. At the elementary level, varied experiences to help fix a concept should be provided.

GOALS

-- Problem solving should be the focus of all school mathematics.

-- Essential mathematical skills should be mastered by all students so that they can function in a technological society.

-- Mathematics programs should take full advantage of the power of calculators and computers at all grade levels.

-- An appreciation of mathematics should be developed.

-- The time spent in the study of mathematics should be increased and used efficiently and effectively.

*A Guide for Curriculum Planning K-12, 4th ed. (DPI, 1983)

MATHEMATICS IN PERSPECTIVE

Now is an exciting time to be in or to be a part of an elementary or secondary institution. The new national recognition of the importance of formal education has provided opportunities for self-examination and to initiate new efforts. Reflecting this environment, the North Dakota Department of Public Instruction has effected a writing of this new Mathematics Curriculum Guide.

As documentation of the interreliance of elementary and secondary education efforts, the Guide presents a curriculum map for elementary as well as secondary school programs. Without a strong elementary school curriculum, delivery of secondary school instruction is delayed and prone to falter. At worst, such secondary education is avoided by the student ill-prepared by inadequate elementary education. With this Guide, there is recognition of the positive potential of the structuring of a map for both elementary and secondary efforts, a must in the present environment.

The North Dakota Department of Public Instruction's perspective of the role for mathematics in the present (the now) is corroborated with an encompassing statement made by the National Council of Teachers of Mathematics. The Council's statement recommends that "Problem solving be the focus of school mathematics, basic skills in mathematics be defined to encompass more than computational facility, mathematics programs take full advantage of the calculators and computers at all grade levels, effectiveness and efficiency be applied to the teaching of mathematics, the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing, more mathematics study be required of all students, mathematics teachers demand of themselves and their colleagues a high level of professionalism, and public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society."

Mathematics is not just a pencil-paper subject; it is an interactive subject requiring communication. Testing, in the usual present day sense, should be done by teachers skilled in observing, questioning, simulating, and structuring group and individual tasks. Development of the total student as a human is the true desired outcome.

The new North Dakota Mathematics Curriculum Guide reflects modern thinking concerning mathematics education. No longer will results or cognitive measures only be valued. Importance will be placed on the process in the context of the total student development approach, where helping students realize their potential in mathematics and develop a positive self-concept are primary goals. The present is now, and today's classroom teachers, educational management, and members of boards of governance have the opportunity for proaction!

MATHEMATICS IN PERSPECTIVE: FUTURE

As we are able to perceive it on the basis of today's technology alone, the future will be drastically different. Methods which were successful in the past may not work in the future, education will not be a function of school alone, and the emphasis will be on lifelong learning of skills.

In the past, people focused on solving problems of the world as it existed. Now and in the future, society will focus on problems in the world as humans have shaped it. Dennis Van Avery in his article "Futuristics and Education" in Educational Leadership, February, 1980, advises us that these problems of tomorrow will require new skills, including a tolerance for multiple interpretations, and the ability to explore and create alternatives. In the 1990s, there will be a need to redefine knowledge, schooling, and education. The emphasis will be on learning how to learn, rather than on learning facts.

Certainly the implication is that instructional alternatives and new skills will be required of the teachers in the future. In this Guide, we have restated and reemphasized curricular goals; and we will need provisions for ongoing inservice by school districts, support for educational change by parents and legislators, and willingness to change and grow by individual teachers.

For all students, a positive attitude toward mathematics will be a critical condition. This positive attitude must be generated by the general attitude and behavior of teachers. Indeed, all students do not need the same mathematics program, nor do they require the same time in learning a skill; but all students should have a good feeling about mathematics and be aware of its important uses in their lives and its valuable function in the world at large.

One of the most difficult challenges for teachers is to prepare their students for vocations which are presently unknown or not even conceptualized. The students of the next decade will be receiving an education which will be the foundation of their successful work effort in the next century. Thus, the incorporation of computer education in mathematics instruction is essential. This Guide will promote such preparation for a future which can only be conjectured (see Appendix B).

PROFESSIONAL INSERVICE AS A COMPONENT

The implication is that instructional alternatives and new skills will be required of teachers in the present and in the future. In this Guide we have restated and reemphasized curricular goals. To accomplish these goals, we will need provisions for ongoing inservice by schools, support of educational change by parents and legislators, and a willingness to change and grow by individual teachers.

In a busy exciting world, teachers are and will be competing for students' attention. The teacher must, therefore, be a well-prepared practitioner who presents ideas with clarity, interesting delivery, and motivating style. With the proliferation of factual data, teachers must be able to locate resources effectively.

The mathematics community needs to build understanding of mathematics and to develop language as the basis for such understanding; strategies for teaching mathematics must be changed to match the way children learn. The scope and sequence of a curriculum that "weeds out" too many students must also be revised. There are ways for all students to learn more mathematics successfully and to develop higher cognitive skills, and we must work with inservice and research to find them. Research and inservice require funding. Legislators, school board members, and the society at large must respond affirmatively!

INDIVIDUAL AND GROUP COOPERATION AS ESSENTIALS

To use this Guide, teachers must be sensitive to the needs of their students and they must be dedicated to the improvement of student learning as their primary professional objective. The rights of students and parents to expect this dedication has correlative responsibilities: a mutual respect and support by the parents of the educational program and a teacher's professional competence, and the acceptance by the student of ultimate personal and active commitment to his/her own learning.

Prominent educators and other social scientists remind us that change is accomplished by individuals not by institutions and that change is a process not an event. Change comes from a growth in skills and feelings. Novel ideas, new curriculum, and fresh strategies are implemented successfully by teachers who have discussed, planned and supported them. The implications for expanded skills, efficient teaching, and flexible programs will be effected informally by dedicated teachers who have shared in the planning for the curriculum of the future.

The professional community and society share a common goal: To bring all citizens to the full realization of their mathematical capacity. This is a complex and delicate task, and it requires the commitment and cooperation of all segments of society. North Dakotans, an independent and self-reliant component of the national and international environment, have, with the Guide, created a tool for learner accomplishment and a societal vehicle for the pursuit of educational excellence.

Dr. Harold Bergquist
Consultant to the North Dakota
Department of Public Instruction

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INTRODUCTION TO THE SECONDARY CURRICULUM

The North Dakota Secondary Mathematics Guide was revised to provide an updated view of what educators across North Dakota feel is necessary in a secondary mathematics curriculum.

The Guide provides four sections that should help the local schools more clearly identify their needs, philosophies and goals for mathematics education. In one section, specific objectives developed by the North Dakota Mathematics Committee are presented to give an overview of perceived statewide mathematics objectives. Two additional sections summarize the National Council of Supervisors of Mathematics (NCSM) Position Paper on Basic Skills and the National Council of Teachers of Mathematics (NCTM) Agenda for Action which give a national perspective to mathematical goals and philosophies. These sections provide both a state and national overview of mathematics that should be useful to local schools in developing mathematics curricula to meet their needs. Also, a section explaining the role of the classroom teacher in dealing with special education students and slower learners has been included. This section should help clarify the expectations of the mathematics teacher in working with these special students.

The Guide identifies seven core courses that provide a foundation for secondary mathematics curriculum. In addition to a course description, each course identifies a list of course objectives. Further, each objective has a sample assessment or comment to help clarify the objectives and help the classroom teacher formulate exercises that evaluate the student's achievement of a particular skill.

Because of the great impact that technology has had on society as well as mathematics, a section addressing the computer and a section focusing upon the use of the calculator have been added to the Guide. An attempt has been made to give some insight into how the computer and calculator can be used effectively in the classroom to improve instruction. In addition, a form has been included to provide a set of guidelines that will aid the schools in choosing appropriate software. A separate section dealing with the use of the calculator in the classroom is also included. The use of the calculator has not been dealt with as an isolated topic in the Guide, but rather, it has been integrated into the curriculum as a computational aid.

Finally, the Bibliography provides an extensive list of resource material for further clarification and expansion of objectives and other topics discussed in the Guide. Although not exhaustive, the Bibliography along with A Guide for Curriculum Planning K-12* should provide a firm foundation for additional help.

*A Guide for Curriculum Planning K-12, 4th ed. (DPI, 1983)

CURRICULUM CONSIDERATIONS FOR THE HANDICAPPED AND SLOW LEARNER

During the last decade, significant legislation has been passed concerning the needs of handicapped students and the responsibility of public schools to address those needs. In summary, federal regulations state that the local school district must provide an appropriate educational program in the least restrictive environment for all identified handicapped students. The slow learner, although not under the federal mandate, may also require similar considerations in programming. In response to the above needs it is necessary to develop guidelines to meet the needs of the target population by providing assistance to regular and special educators.

The area of mathematics is of special concern to regular and special educators. Not only is it a major tool area, but is also an area where the handicapped learner is frequently mainstreamed. The provision of an appropriate education program for the handicapped learner should provide a challenging math program at the student's ability level. Program planners will need to give special consideration to the relevancy of mathematics curriculum as it relates to each student's unique needs.

Federal and state regulations require that all school districts identify handicapped students. This process may also identify other students who require special considerations. A building level team approach is one method of identifying and providing for those students who do not categorically qualify for an area of special education. Team membership could include building administrator, regular and special educators, parents, students, and individuals from other agencies.

Students identified as handicapped require individualized education programs. The construction of the IEP is a team approach. This team is responsible for determining whether a student is able to participate in regular math classes without modifications, in regular math classes with modifications or if an alternative program needs to be developed. Building level teams may follow similar procedures with nonhandicapped students.

If the team decision is to enroll the student in the regular classroom with modifications, these modifications will be limited only by the creativity of the team members. Some examples might include: 1) altering time and quantity requirements, 2) use of learning packets, 3) use of audiovisual aids, 4) computer assisted instruction, 5) test-taking options, 6) alternate materials such as texts, 7) peer tutoring, 8) team teaching, 9) contracting, 10) selecting the most relevant math objectives, etc. Teachers making these modifications may want to consult with building administrator and special education personnel to explore grading options.

Should the team decide that the student would be unable to function in the regular classroom with modifications, an alternative program will be developed by the team. Those developing alternative programs may wish to refer to scope and sequence references such as district curriculum guides, commercial scope and sequence charts, and North Dakota Department of Public Instruction Mathematics Curriculum Guide K-8 (1986). Although a team approach may be used, the primary responsibility for instruction of mainstreamed math students lies with the classroom teacher.

The following persons were involved in writing this section. Their assistance is appreciated.

James Stenehjem, Principal
Watford City High School

Wayne Triska, Vision Impaired
Bismarck Public Schools

Twila Jarrett, EMH Teacher
Bismarck Public Schools

Joan Bonsness, LD Teacher
Central Campus High School, Minot

Kathryn Plowman, LD Teacher
Oliver/Mercer Special Education Unit, Center

Neil Pederson, Math Teacher
Carrington High School

Betty Vibeto, Program Coordinator
Souris Valley Special Education Unit, Minot

Linda Hartman, LD Teacher
Central High School, Grand Forks

Connie Hovendick, ED Teacher
Lake Region Special Education Unit, Devils Lake

Chris Strom, Program Coordinator
Lake Region Special Education Unit, Devils Lake

Donald Boehmer, Director
Lake Region Special Education Unit, Devils Lake

Judi Bryl, Business Manager
Lake Region Special Education Unit, Devils Lake

GENERAL COURSE OBJECTIVES

ALGEBRA I

To develop the algebraic skills necessary for future study of mathematics.

To develop the ability to solve problems algebraically.

To develop the ability to apply algebra to everyday situations--social, technical, academic.

GEOMETRY

To develop an understanding of geometric concepts.

To develop the ability to think critically.

To develop the geometry necessary for future study of mathematics.

To develop an appreciation for geometry and its importance in the world.

To develop the ability to apply geometry to everyday situations--social, technical, academic.

ALGEBRA II

To develop an appreciation for algebra.

To develop an extension of the algebraic skills developed in Algebra I necessary for future study of mathematics.

To develop the ability to solve problems algebraically.

To develop the ability to apply algebra to everyday situations--social, technical, academic.

To develop an ability to solve problems.

TRIGONOMETRY

To develop an understanding of angular-linear relationships.

To develop an appreciation for the many applications of periodic functions.

To develop knowledge and techniques for further study of mathematics.

To develop career skills needed for employment.

ANALYTIC GEOMETRY

To develop an appreciation for and understanding of algebraic descriptions of geometric ideas.

To develop an understanding of planar and spatial relationships.

To develop techniques needed for further study of mathematics.

PROBABILITY AND STATISTICS

To develop the ability to use statistical and probabilistic reasoning.

To develop a critical attitude toward statistics found in everyday life.

To develop a background for the study of probability and statistics in college.

MATHEMATICAL APPLICATIONS

Problem Solving

To develop a strategy to effectively solve practical problems that come from "real-world" applications.

To develop the ability to use a calculator to help solve problems.

Informal Geometry

To develop an understanding of geometric concepts.

To develop reasoning power.

To develop an appreciation for geometry and its importance in the world.

To develop the ability to apply geometry to everyday situations--social, technical.

Everyday Statistics

To develop the ability to use statistical and probabilistic reasoning.

To develop a critical attitude toward statistics found in everyday life.

Mathematics of Finance

To develop the ability to utilize mathematics to make sound consumer decisions.

To develop confidence in the use of mathematics in finance.

To develop the mathematics of finance necessary to effectively interact with society.

To develop mathematical finance skills necessary for employment.

SUGGESTED COURSE SEQUENCES

Algebra contains many of the math concepts and abstract thinking skills necessary for a well-rounded education. Therefore, algebra is highly recommended as one of the courses a student should complete before graduation from high school.

Since all ninth grade students do not possess the maturity and/or capability to handle algebra as a traditional one-year course, school districts may want to consider other course options that are more suitable for their students. One option might include an Introductory Algebra course. This one-year course includes many of the objectives listed in the course description of Algebra I in this Guide.* An Introductory Algebra text used in conjunction with an Algebra I text of the same publisher should provide a good foundation of algebra skills.

A second option might be to divide Algebra I into a two-year course, however one credit must be given for each mathematics course successfully completed. These options do not provide all the necessary math skills a student should have before graduation. The completion of a third mathematics course such as an applications course is important for the student to graduate with adequate mathematics skills.

Two other recommendations are useful regarding an adequate training in mathematics. First, each student should take a mathematics course their junior or senior year to update and expand previously learned skills. Second, college bound students should take four years of high school mathematics to facilitate the transition from high school to college.

College Preparatory

Program I

8th Grade	Algebra I
9th Grade	Geometry
10th Grade	Algebra II
11th Grade	Trig/Analytic Geometry
12th Grade	Probability and Statistics and other options

Program II

9th Grade	Algebra I
10th Grade	Geometry
11th Grade	Algebra II
12th Grade	Trig/Analytic Geometry

Program III

9th Grade	Introductory Algebra
10th Grade	Algebra I
11th Grade	Geometry
12th Grade	Algebra II

*The objectives identified as appropriate for an Introduction to Algebra course, are marked with an asterisk in the Algebra I course objectives.

Noncollege Preparatory

Program IV	
9th Grade	Introductory Algebra
10th Grade	Algebra I
11th Grade	
or	Mathematical Applications
12th Grade	
Program V	
9th Grade	Algebra I
10th Grade	
11th Grade	
or	Mathematical Applications
12th Grade	
Program VI	
9th Grade	Introductory Algebra
10th Grade	Algebra I
11th Grade	Mathematical Applications
12th Grade	Geometry or Algebra II
Program VII	
9th Grade	Algebra I
10th Grade	
11th Grade	
or	Geometry or Algebra II
12th Grade	
Program VIII	
9th Grade	Algebra I - first half
10th Grade	Algebra I - second half
11th Grade	Mathematical Applications
12th Grade	Geometry or Algebra II
Program IX	
9th Grade	Algebra I - first half
10th Grade	Algebra I - second half
11th Grade	
or	Mathematical Applications
12th Grade	

SECONDARY MATHEMATICS
COURSE OBJECTIVES

ALGEBRA I

I. Introduction to algebra

A. Numerals and expressions

1. Symbols of grouping
2. Order of operations
3. Properties of real numbers
4. Absolute value
5. Exponents and factors

B. The number line

1. Comparing two numbers
2. Graphing sets of numbers

C. Operations with signed numbers
D. Translating words into symbols
E. Formula evaluation by replacement method

II. Linear equations and inequalities in one variable

A. Solving linear equations
B. Solving linear inequalities
C. Graphical interpretations of linear expressions
D. Applications

III. Linear equations and inequalities in two variables

A. Cartesian coordinate system
B. Domain and range
C. Relations and functions
D. Solution sets

E. Graphing linear equations and inequalities

1. Slope - intercept
2. Intercept form
3. Two-point form
4. Table of ordered pairs

F. Methods of solving linear systems

1. Graphing
2. Addition and/or subtraction
3. Substitution

G. Determining equations of lines
H. Direct variation
I. Applications

IV. Rational expressions and equations

A. Law of exponents

1. Positive
2. Negative
3. Zero

B. Simplification of rational expressions
C. Multiplication and division
D. Addition and subtraction
E. Ratio and proportion
F. Changing repeating decimals to rational expressions
G. Fractional equations
H. Applications

V. Polynomials

A. Applications of basic operations
B. Factoring
C. Special products

VI. Irrational expressions and equations

A. Radical expressions
B. Simplifying radicals
C. Fundamental operations with radicals
D. Solving radical equations

VII. Quadratic equations

A. Methods of solving

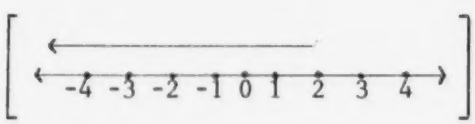
1. Factoring
2. Completing the square
3. Quadratic formula

B. Graphing quadratic equations
C. Inverse variation

ALGEBRA I

OBJECTIVES	EXAMPLES OR COMMENTS
The student should be able to:	
I. Introduction to algebra	
* A. Numerals and expressions	
* 1. & 2. Evaluate expressions using symbols of grouping and order of operations.	1. & 2. Evaluate: $8 + 2[14 - 2(3 + 5) \div 4]$ [28]
* 3. Use properties of real numbers.	3. $3(4 + 5) = 12 + 15$ demonstrates what property of real numbers? [distributive]
* 4. Evaluate absolute value.	4. Evaluate: $\lvert -7 \rvert$ [7]
* 5. Evaluate expressions involving exponents.	5. Evaluate: $15 + 2^3 - 2 \cdot 3^2$ [5]
* B. The number line	
* 1. Compare two numbers using the symbols $>$, $<$, or $=$.	1. Write: $3 > 1$ in words [Three is greater than one]
* 2. Map subsets of the real numbers onto the number line.	2. Graph on the number line: $\{-2, 1/2, \sqrt{2}, 3\}$
* C. Operations with signed numbers	
* 1. Add, subtract, multiply and divide signed numbers.	1. Evaluate: $-2(-3)$ [6]
* D. Translating words into symbols	
* 1. Change the words into symbols.	1. Write: -2 is less than 5, in mathematics symbolic notation. [$-2 < 5$]

*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

* E. Formula evaluation by replacement method	
* 1. Evaluate formulas relating to geometry, business, and science.	1. Find C in $C = \pi r^2$ if $\pi \cong \frac{22}{7}$ and $r = 14$ [$C \cong 616$]
II. Linear equations and inequalities in one variable	
* A. Solving linear equations	
* 1. Solve linear equations.	1. Solve for x: $3x + 2 = 11$ [3]
* B. Solving linear inequalities	
* 1. Solve linear inequalities.	1. Solve for x: $3x < 6$ [$x < 2$]
* C. Graphical interpretations of linear expressions	
* 1. Graph the solution sets of open sentences on number line.	1. Graph the solution set of $3x < 6$ 
* D. Applications	
* 1. Analyze verbal problems, write a mathematical sentence, find the solution set, and then check the answer.	1. The clearance on the sides of a bearing is .034 cm. The clearance on one side is supposed to be .007 cm more than twice the clearance on the other side. Find the two clearances. [.009 cm and .025 cm]

*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

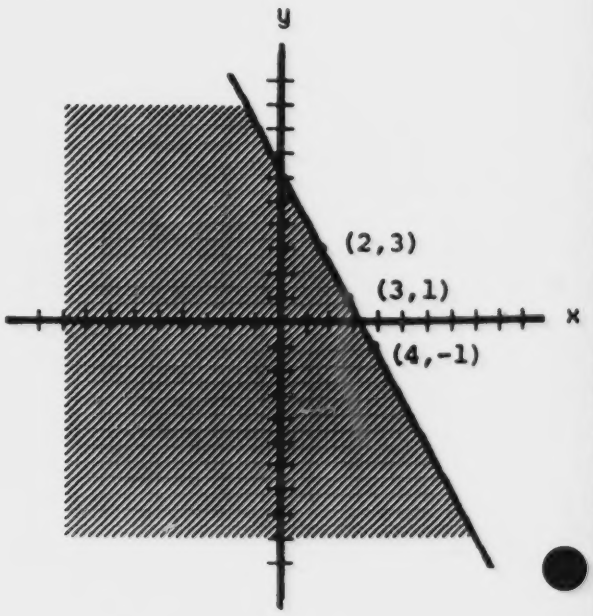
III. Linear equations and inequalities in two variables

* A. Cartesian coordinate system	
* 1. Demonstrate the relationship between ordered pairs and points on a plane.	1. Graph: (2,3) and (-1,-4)
B. Domain and range	
1. State the domain and range of different functions.	1. Given: $f(x) = 2x - 3$, $x \in \{0,1,4\}$ find the range and the domain. Domain = $\{0,1,4\}$ Range = $\{-3,-1,5\}$
C. Relations and functions	
1. Distinguish between relations that are functions and those which are not functions.	1. Is the following relation a function: $\{(-2,1), (-1,-2), (0,0), (2,2)\}$ [yes]
* D. Solution sets	
* 1. Find the solution set of a linear equation.	1. Find the solution set for: $2x + 3y = 12$, $x \in \{0,3,6\}$ $[\{(0,4);(3,2);(6,0)\}]$
* E. Graphing linear equations and inequalities	
1. Graph a linear equation in two variables using the slope-intercept method.	1. Graph: $2x + y = 2$, using the slope-intercept method. $\left[\begin{array}{l} m = \frac{-2}{1} \\ y\text{ - intercept} = (0,2) \end{array}\right]$
2. Graph a linear equation in two variables using the intercept method.	2. Graph: $2x + 3y = 6$, using the intercept method. [i.e., plot (0,2) and (3,0)]

*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

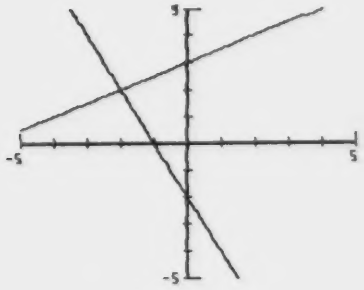
* 3.	Graph a linear equation in two variables using the two-point method.	3. Graph: $2x - 3y = 6$, using a two-point method. [e.g., plot (6,2) and (3,0)]
4.	Graph a linear inequality in two variables using a table of values.	4. Graph. $2x + y \leqslant 7$

x	y
2	3
3	1
4	-1



* F. Methods of solving linear systems

* 1.	Find the solution set of a system of linear equations by the graphing method.	1. Use the graphing method to find the solution set of the following: $x - 2y = -6$ and $2x + y = -2$ [(-2,2)]



*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

* 2. Find the solution set of a system of linear equations by the addition/subtraction method.	2. Use the addition/subtraction method to solve: $x - 2y = -6$ $2x + y = -2$ [(-2, 2)]
* 3. Find the solution set of a system of linear equations by the substitution method.	3. Use the substitution method to solve: $x - 2y = -6$ $2x + y = -2$ [(-2, 2)]
G. Determining equations of lines	
1. Determine the equation of a line from given data.	1. Find the equation of the line which passes through (5, 3) and (-7, 6). [$x + 4y = 17$]
H. Use direct variation to solve problems	H. Given that x varies directly as y, and that $x = 30$ when $y = 3$ find: The constant of variation and the value of x when the value of y is 4. [$10 = K$ $x = 40$]
* I. Analyze verbal problems	I. You have 40 coins consisting of nickels and dimes which have a value of \$2.85. How many of each kind are there? [23 nickels and 17 dimes]
IV. Rational expressions and equations	
* A. Laws of exponents	
* 1. Simplify rational expressions with positive exponents.	1. Simplify: $\frac{x^3}{x}$ [x^2]

*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

	2. Simplify rational expressions with negative exponents.	2. Simplify: $2x^{-5} \cdot x^{3}$ $\left[\frac{2}{x^{2}}\right]$
	3. Simplify rational expressions with zero exponents.	3. Simplify: $\frac{x^{3}}{x^{2}y^{0}}$ [x]
* B.	Simplify rational expressions	B. Simplify: $\frac{16x^{3}y^{4}}{56xy^{5}}$ $\left[\frac{2x^{2}}{7y}\right]$
* C.	Multiply and divide rational expressions	C. Multiply and simplify: $\frac{3x}{y^{2}} \cdot \frac{5y^{3}}{3x}$ [5y]
	D. Add and subtract rational expressions	D. Simplify: $\frac{(x+1)}{x} - \frac{x}{(x+1)}$ $\left[\frac{2x+1}{x(x+1)}\right]$
* E.	Solve problems using ratio and proportion	E. Solve for x: $\frac{5}{x+3} = \frac{3}{2x-1}$ [x = 2]
	F. Change repeating decimals to rational expressions	F. Change $.\overline{72}$ to a fraction. $[\frac{8}{11}]$
	G. Solve fractional equations	G. Solve for b: $\frac{3}{10} = \frac{1}{2} - \frac{1}{b}$ [b = 5]

*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

* H. Analyze verbal problems	H. The State Game and Fish Department stocked Devils Lake with walleye and northern in the ratio of five to three. If 2,400 fish were released in the lake, how many of each kind were released? [walleye = 1,500 northern = 900]
V. Polynomials	
* A. Apply basic operations	A. Simplify: $2x(3x - 5) + 4x(x + 2)$ [$10x^2 - 2x$]
* B. Factor polynomials	B. Factor: $10x^2 + 13x - 3$ [$(2x + 3)(5x - 1)$]
* C. Find special products of polynomials	C. Find the product: $(a - b)(a + b)$ [$a^2 - b^2$]
VI. Irrational expressions and equations	
* A. Evaluate radical expressions by the use of calculators or tables	A. Approximate $\sqrt{3}$ to the nearest hundreth. [1.73]
* B. Simplify radicals	B. Simplify: $\sqrt{12}$ [$2\sqrt{3}$]
C. Perform basic operations with radical expressions	C. Simplify: $\sqrt{8} + \sqrt{27} + \sqrt{18}$ [$5\sqrt{2} + 3\sqrt{3}$]
D. Solve radical equations	D. Solve for x: $\sqrt{x} - 4 = 7$ [121]

*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

VII. Quadratic equations

* A. Methods of solving

* 1. Solve quadratic equations by factoring.

1. Solve by factoring:

$x^2 - 6x + 8 = 0$

[x = 2,4]

2. Solve quadratic equations by completing the square.

2. Solve by completing the square:

$x^2 + 8x + 4 = 0$

$[x = -4 \pm 2\sqrt{3}]$

3. Solve quadratic equations by using the quadratic formula.

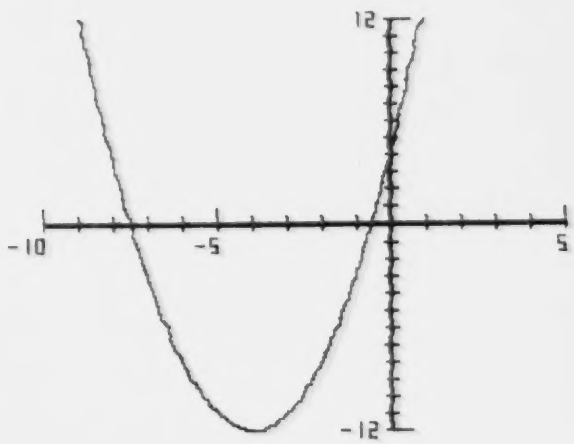
3. Solve by using the quadratic formula:

$2x^2 + x - 3 = 0$

$[x = -\frac{3}{2}, 1]$

B. Graph a quadratic function

B. Graph: $x^2 + 8x + 4 = y$



C. Use inverse variation to solve problems

C. If x varies inversely as y^2, and x = 25 when y = 2, find x when y = 5.

[x = 4]

*Indicates objectives that should be used as objectives for an "Introduction to Algebra" course.

I. Concepts of sets

II. Real numbers

III. Language of geometry

IV. Logic and proofs

- A. Inductive and deductive reasoning
- B. Direct and indirect proofs

V. Angles

- A. Measurement of angles
- B. Classification of angles

VI. Triangles

- A. Congruence
- B. Properties of special triangles
- C. Geometric inequalities
- D. Ratio and proportion
- E. Similarity
- F. Right triangle trigonometry

VII. Parallels and perpendiculars

VIII. Quadrilaterals and other polygons

IX. Circles

X. Constructions

XI. Measurement

- A. Perimeter and area
- B. Surface area and volume

XII. Coordinate geometry

XIII. Transformations (Optional)

OBJECTIVES	EXAMPLES OR COMMENTS
The student should be able to:	
I. Concepts of sets	
A. Apply the concept of set notation, including subsets and the operations of union and intersection, to geometric figures.	A. A separate unit on sets is not implied but can be integrated throughout the text.
B. Define a geometric figure as a set (locus) of points.	B. The intersection of two lines is a point.
II. Real numbers	
A. Use the basic axioms and theorems on real numbers in writing proofs and solving problems.	A. Properties such as reflexive, symmetric, and transitive.
III. Language of geometry	
A. List the undefined terms: point, line, and plane.	A. Appreciate the need for undefined terms in an axiomatic system.
B. Define and/or illustrate basic geometric figures.	B. Definitions of geometric figures such as: line segment, skew lines, types and parts of triangles, quadrilaterals, angles.
IV. Logic and proofs	
A. Use and differentiate between inductive and deductive reasoning.	A. Inductive: It rained Monday, Tuesday, Wednesday. Conclusion: It will rain on Thursday. Deductive: Premise: All students are happy; John is a student. John is happy.
B. Recognize and write direct and indirect proofs.	B. Prove: If two lines form equal corresponding angles with a transversal, the lines are parallel.

V. Angles

A. Determine the measure of a given angle.

A. Estimate and then measure an angle.

B. Classify angles.

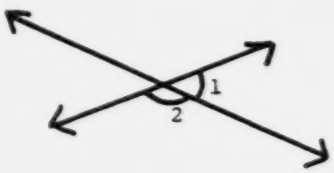
1. Identify angles as obtuse, acute, or right.

1. Classify the following angle as acute, obtuse, or right:

[acute]

2. Identify pairs of angles as adjacent and/or supplementary.

2. Classify the following angles as adjacent and/or supplementary:

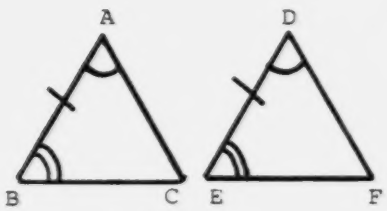


[adjacent and supplementary]

VI. Triangles

A. State the conditions necessary to show two triangles to be congruent, i.e. SAS, ASA, SSS, HL, AAS, HL.

A.



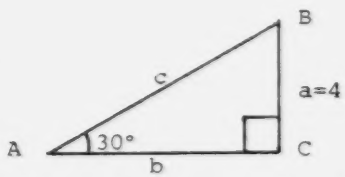
Is triangle ABC congruent to triangle DEF?

[Yes, by ASA]

B. Special triangles

1. State the relationships of the sides of the 45°- 45°- 90° and 30°- 60°- 90° triangles.

1.



Using the given triangle, determine the values for b, c, and B.

[$b = 4\sqrt{3}$, $c = 8$, $B = 60°$]

2. State and use the special properties of isosceles and equilateral triangles.

2. In the isosceles triangle, what is the relationship of the bisector of the vertex angle, the altitude to the base and the median to the base?

[they are equivalent]

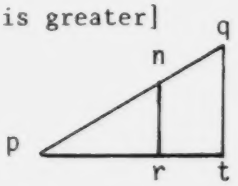
C. State and use the basic inequalities relating to sides and angles of a triangle.

C. How is the sum of the lengths of two sides of a triangle related to the length of the third side?

[it is greater]

D. Use ratios and proportions and their properties in solving problems and writing proofs.

D.



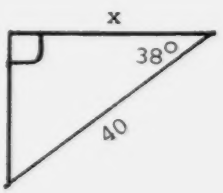
If $\frac{pn}{pq} = \frac{pr}{pt}$, then prove $\overline{nr}$ is parallel to $\overline{qt}$.

E. Define similarity of triangles, prove similarity theorems and use properties of similar triangles to solve problems.

E. Prove: If two angles of one triangle are congruent to two angles of a second triangle, the triangles are similar.

F. Recognize and use the sine, cosine, and tangent ratios as related to the right triangle.

F. Find x



[31.5]

VII. Parallels and perpendiculars

A. State and apply methods of proving lines parallel and perpendicular and conclusions which follow from having lines parallel and perpendicular.

A. Prove the sum of the measures of the angles of a triangle is 180°.

VIII. Quadrilaterals and other polygons

A. Identify the various types of quadrilaterals and apply triangle methods to them.

A. Prove that the triangles formed by the diagonals of a rectangle are congruent.

B. Identify similar polygons, regular polygons, and their parts.	B. Define a regular polygon.
IX. Circles	
A. Solve problems involving arcs and angles related to the circle and problems involving line segments related to the circle.	A. If a line passes through a point P on a circle and is perpendicular to a radius of the circle at P, the line is a tangent line to the circle at P.
X. Constructions	
A. Make simple constructions with compass and straightedge and write proofs of them.	A. Using a compass and a straightedge, bisect a line segment.
XI. Measurement	
A. Find areas and perimeters of triangles, trapezoids, parallelograms, rectangles, squares, circles, and regular polygons.	A. Given a triangle with sides 3 m, 4 m, and 7 m, find the perimeter. [14 m]
B. Find volumes and surface areas of spheres, cones, cylinders, prisms, and pyramids.	B. Find the volume of a right cylinder with height = 7 cm and radius of base = 12 cm. [3166.7 cm^3]
XII. Coordinate geometry	
A. Use coordinate geometry to describe points and lines in a plane.	A. Find the midpoint of the line segment determined by (3,2) and (15,8). [(9,5)]
XIII. Transformations (Optional)	
A. Locate images of geometric figures by translation, reflection, and rotation.	A. Use tinted plexiglass to help visualize concepts.

ALGEBRA II

I. Use of axioms, theorems and their converses in determining rules of computations

II. Solving first degree equations and inequalities with one variable

- A. Equations
- B. Inequalities
- C. Applications

III. First degree equations and inequalities

- A. Graphs of linear equations and inequalities
- B. Determining equations of a line
- C. Systems of linear equations
- D. Graphing systems of linear equations and inequalities
- E. Applications

IV. Polynomials

- A. Operations
- B. Factoring

V. Real numbers

- A. Rational
- B. Irrational
- C. Simplify radicals
- D. Operations

VI. Complex numbers

- A. Imaginary numbers
- B. Operations

VII. Exponents, exponential functions, and logarithmic functions

- A. Extend laws of exponents
- B. Exponential functions
- C. Inverse functions
- D. Properties of logarithms
- E. Logarithmic functions
- F. Applications

VIII. Quadratic equations and inequalities

- A. Direct and inverse variation
- B. Quadratic formula and completing the square
- C. Graphing
- D. Conic sections
- E. Solving systems

IX. Sequences and series

A. Arithmetic
B. Geometric
C. Binomial theorem

X. Matrices and determinants

A. Operations
B. Solution set for systems

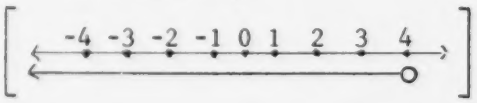
XI. Examining roots of higher degree equations

A. Finding rational roots
B. Factor theorem and synthetic division

XII. Permutations, combinations, and probability

A. Definitions and formulas
B. Properties of probability
C. Applications

ALGEBRA II

OBJECTIVES	EXAMPLES OR COMMENTS
The student should be able to:	
I. Use of axioms, theorems and their converses in determining rules of computations	
A. Use algebraic axioms and theorems.	A. Simplify: $2(3x + 5)$ [$6x + 10$]
II. Solving first degree equations and inequalities with one variable	
A. Solve equations with one variable of first degree and specify various subsets by the roster method.	A. Determine: $\{x: x + 2 = 2x - 7\}$ and x is an element of the integers [$\{9\}$]
B. Solve inequalities with one variable of first degree and graph the solution set.	B. Determine: $\{y: y + 2 > 3y - 6$ and y is an element of the real numbers$\}$ 
C. Analyze verbal problems, write a mathematical sentence, find the solution set and then check.	C. The equal sides of an isosceles triangle are each 5 cm less then twice the base. If the perimeter is at most 60 cm, find the maximum length of the base. [14 cm]
III. First degree equations and inequalities	
A. Graph linear equations and inequalities.	A. Graph the line whose equation is $2x - 3y = 6$.

B. Determine the equations of the lines which satisfy given conditions.

B. Find the equation of the line passing through the point (7, -3) and having a slope of 1/2, using the form $Ax + By = C$ with A, B, C elements of the integers.

[$\{(x,y): \ x - 2y = 13\}$]

C. Solve systems of two simultaneous linear equations and state whether they are consistent or inconsistent.

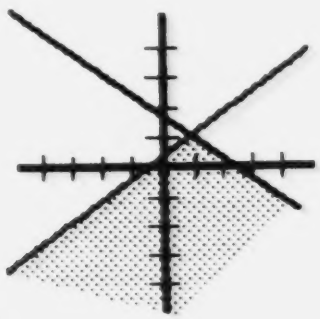
C. Using the substitution method determine all ordered pairs (x,y) that will simultaneously satisfy the two linear equations $2x + y = 4$, $y = 6x - 12$ and state whether the system is consistent or inconsistent.

[$\{(2,0)\}$; system is consistent]

D. Solve two simultaneous linear inequalities by graphing.

D. Graph the following system:

$2x + 3y \leq 5$
$3x - 4y \geq -1$



IV. Polynomials

A. Perform operations.

A. Add $3x - 2y$ and $2x - 4y$

[$5x - 6y$]

B. Factor

B. Factor: $27k^3 - 1$

[$(3k - 1)(9k^2 + 3k + 1)$]

V. Real numbers

A. & B. Identify rational and irrational numbers.

A. & B. Identify the following as rational or irrational:

1. $\sqrt{4}$

[rational]

2. $\sqrt{18}$

[irrational]

C. Simplify radicals.

C. Simplify: $\sqrt{48}$

$[4\sqrt{3}]$

D. Perform operations.

D. Simplify the following:

$2\sqrt{2} + 4\sqrt{2}$

$[6\sqrt{2}]$

VI. Complex numbers

A. Represent complex numbers in the a + bi form.

A. Represent $4 + \sqrt{-9}$ in the form a + bi

[4 + 3i]

B. Perform operations.

B. Simplify and place the following in standard form:

(3 + 2i) - (6 - 4i)

[-3 + 6i]

VII. Exponents, exponential functions, and logarithmic functions

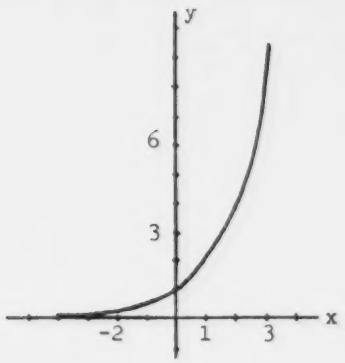
A. Perform operations involving exponents.

A. Simplify the following:

$(x^{-3})^{2/3}$ $\left[\frac{1}{x^2}\right]$

B. Graph exponential functions.

B. Graph: $y = 2^x$



C. Find the inverse of a function.

C. Find the inverse of:

$y = a^x$

$[x = a^y \text{ or } y = \log_a x]$

D. Utilize properties of logarithms.

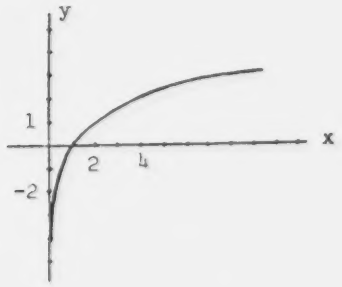
D. Solve the following:

$\log_2 16 = x$

$[x = 4]$

E. Graph logarithmic functions and other inverse functions.

E. Graph: $y = \log_2 x$



F. Solve equations using logarithms.

F. How long will it take for $1,000 invested at 8 percent, compounded semiannually, to grow to $3,000?

[14 years]

VIII. Quadratic equations and inequalities

A. Solve functions which are in the form of a direct variation or an inverse variation.

A. For a fixed amount of a certain gas, pressure (P) varies directly as the temperature (T) of the gas and inversely as the volume (V) of the gas.

Complete the formula $P =$ _____ using m to denote the constant.

[$P = \frac{mT}{V}$]

B. Solve quadratic equations or inequalities by using the quadratic formula, the method of completing the square, or factoring.

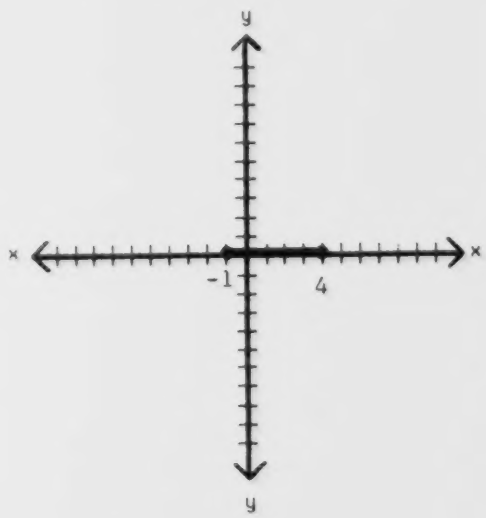
B. Solve using any method:

$2x^2 + 4x - 5 = 0$

[$x = -1 \pm \frac{1}{2}\sqrt{14}$]

C. Solve quadratics by graphing.

C. Solve $x^2 - 3x - 4 \leqslant 0$ by graphing.

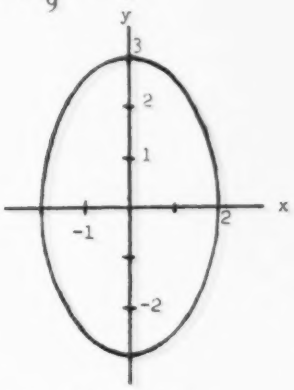


$\{-1 \leqslant x \leqslant 4\}$

D. Recognize conic sections by their equations and sketch the graph.

D. Sketch:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



E. Write the solution set of a given system of linear or quadratic equations.

E. Solve the system:

$$x^2 + y^2 = 25$$

$$x^2 - y^2 = 7$$

$[\{(4,3),(4,-3),(-4,3),(-4,-3)\}]$

IX. Sequences and series

A. Utilize arithmetic formulas.

A. Find the sum of the first 100 natural numbers.

[5050]

B. Utilize geometric formulas.

B. Insert three geometric means between 1 and 81.

[3, 9, 27]

C. Apply the binomial theorem.

C. Find the fourth term of $(a + b)^{10}$.

$[120a^7b^3]$

X. Matrices and determinants

A. Perform operations.

A. Given $A = \begin{bmatrix} 2 & -1 \\ 6 & 3 \end{bmatrix}$ find 3A.

$[3A = \begin{bmatrix} 6 & -3 \\ 18 & 9 \end{bmatrix}]$

B. Find the solution set for a system of n-equations with n-unknowns, using matrices.

B. Find the solution set for the system:

$$\begin{aligned} x + y + z + w &= 8 \\ x - y \quad - w &= -2 \\ y + z + 2w &= 9 \\ x \quad + 2z \quad &= 6 \end{aligned}$$

[y = 1 x = 2
z = 2 w = 3]

XI. Examining roots of higher degree equations

A. Find all possible rational roots of a polynomial equation with integral coefficients.

A. Find all possible rational roots of:

$y^3 - 2y^2 + 5y - 4 = 0$

[+1, -1, +2, -2, +4, -4]

B. Apply the factor theorem or synthetic division to determine the roots of a polynomial equation.

B. Using synthetic division find all roots of:

$y^3 - 2y^2 + 5y - 4 = 0$

$[1, \frac{1}{2} + \frac{\sqrt{15}i}{2}, \frac{1}{2} - \frac{\sqrt{15}i}{2}]$

XII. Permutations, combinations, and probability

A. Apply definitions and formulas.

A. Determine the number of ways five students can be seated in a row of five desks:

[5! or 120]

B. Apply the properties of probability.

B. If two dice are rolled, what is the probability of them both showing the same number?

$[\frac{1}{6}]$

C. Recognize the appropriate formula in order to solve problems.

C. Ten beads are numbered one to ten and placed in a box. If two beads are selected at random and simultaneously from the box, what is the probability that their sum is ten?

$[\frac{4}{45}]$

TRIGONOMETRY

Introduction

The following outline presumes that trigonometry will be taught as a separate semester course. Schools that teach trigonometry as portions of other courses will have to make the appropriate adaptations. The suggested course introduces the material in terms of the right triangle followed by the unit circle. Some instructors may prefer to reverse the order of introduction. Students should complete the course being proficient with both approaches. The use of calculators should be encouraged and emphasized. A computer with appropriate software can be very helpful in illustrating graphical concepts.

I. Introduction to trigonometric functions

- A. Degree measure of angles
- B. Trigonometric functions
- C. Identities

II. Trigonometric functions as circular functions

- A. Radian measure
- B. Applications using radian measure
- C. Circular functions
- D. Pythagorean identities

III. Graphs

- A. Graphs of trigonometric functions
- B. Generalized sine and cosine functions
- C. Addition of ordinates

IV. Trigonometric identities

- A. Proving identities
- B. Sum and difference identities
- C. Double and half angle identities
- D. Other identities

V. Inverse trigonometric functions

- A. Definitions
- B. Graphs
- C. Computations
- D. Trigonometric equations

VI. Problem solving with triangles

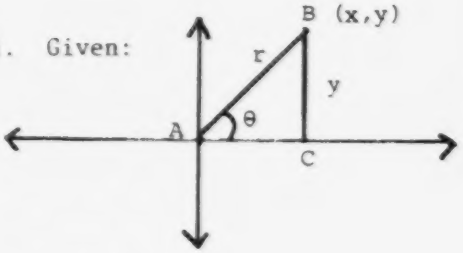
- A. Right triangles
- B. Oblique triangles

VII. Vectors in the plane

A. Notation and operations
B. Applications

VIII. Complex Numbers

A. Definition and notation
B. Operations
C. DeMoivre's Theorem

OBJECTIVES	EXAMPLES OR COMMENTS
The student should be able to:	
I. Introduction to trigonometric functions	
A. Review degree measure of angles.	A. Draw an angle of 23°.
B. Define trigonometric functions.	
1. Define the six trigonometric functions as ratios of the sides of a right triangle.	1. Given:  Define sin θ. [sine θ = sin θ = $\frac{y}{r}$ = $\frac{\text{side opposite}}{\text{hypotenuse}}$]
2. Learn the trigonometric function values of 30°, 45°, and 60° angles.	2. Without using a table, determine the six trigonometric function values of θ = 30°.
C. Derive identities.	
1. Derive basic identities for tan θ and cot θ.	1. Derive tan θ = $\frac{\sin \theta}{\cos \theta}$ using the definitions of the trigonometric functions.
2. Derive reciprocal identities.	2. Derive the following reciprocal identity: csc θ = $\frac{1}{\sin \theta}$
3. Derive cofunction identities.	3. Derive the following cofunction identity: sin θ = cos (90° - θ)

II. Trigonometric functions as circular functions

A. Define radian measure and its relationship to degree measure.

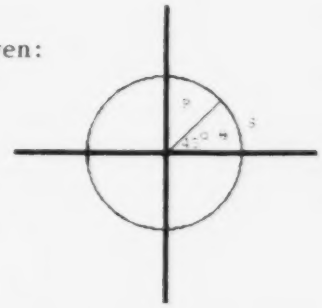
A. 180° is equal to how many radians?

(π radians)

B. Use applications of radian measure.

1. Determine arc length and area of a sector.

1. Given:



find arc length and area of sector if R = 10.

$[\frac{5\pi}{2}, \frac{25\pi}{2}]$

2. Determine the relationship between angular and linear velocity.

2. Two pulleys of diameter 50 cm and 25 cm respectively are connected by a belt, the larger pulley rotates 800 times per minute, find the speed of the belt in centimeters per second and the angular velocity of the smaller pulley.

$[\frac{2{,}000\pi}{3}$ cm/sec, 1,600 rpm]

C. Use circular functions.

1. Define the trigonometric functions in terms of coordinates of a point on the unit circle.

1. Define: sin θ.

[sin θ = y where (x,y) is a point on unit circle θ units from (1,0)].

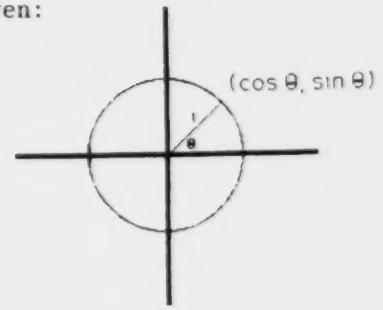
2. Describe the domain and range of each trigonometric function.

2. Describe the domain and range of sin θ.

[Domain of sin θ consists of all real numbers. Range of sin θ consists of all reals between -1 and 1 inclusive.]

D. Derive pythagorean identities.

D. Given:



Derive: $\sin^2 \theta + \cos^2 \theta = 1$

$\tan^2 \theta + 1 = \sec^2 \theta$

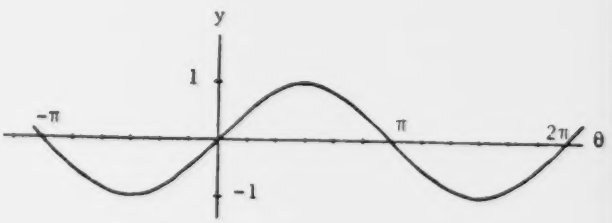
$\cot^2 \theta + 1 = \csc^2 \theta$

III. Graphs

A. Graph the six basic functions.

1. Sketch the graph.

1. Graph: $y = \sin \theta$.



2. Determine whether each function is even or odd.

2. Is sin θ an odd or even function?

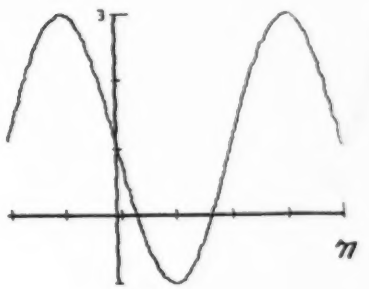
3. Determine the period of each function.

3. What is the period of the function y = sin θ?

B. Generalize sine and cosine functions.

1. Graph functions of the form:
   $y = a \sin b(\theta + c) + d$
   and
   $y = a \cos b(\theta + c) + d$.

1. Graph:
   $y = 2 \sin 2(\theta + \frac{\pi}{2}) + 1$



2. Define and use the concepts of period, amplitude, phase shift, and vertical shift.

2. Given:
   $y = 3 \sin 2 (\theta + 2) + 3$
   make several changes in each constant, changing only one constant at a time. Graph each function. Describe how each change of constant affects the graph. A microcomputer with appropriate software would be invaluable and should be utilized to increase the number of examples students can examine in a reasonable amount of time.

C. Graph by addition of ordinates method.

C. Graph: $\sin \theta + \cos \theta = y$.

Additional examples could be profitably examined with a microcomputer.

IV. Trigonometric identities

A. Develop techniques for proving identities.

A. Verify:

$\cot \theta + 1 = \csc \theta (\cos \theta + \sin \theta)$

B. Derive sum and difference identities.

B. Verify:

$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

C. Derive double angle and half angle identities.

C. Verify:

$\sin 2\theta = 2 \sin \theta \cos \theta$

D. Develop skills in proving other identities.

D. Verify:

$$\frac{2 \tan \theta}{1 + \tan^2 \theta} = \sin 2\theta$$

V. Inverse trigonometric functions

A. Apply definitions.

1. Examine inverse trigonometric relations.

1. Show:

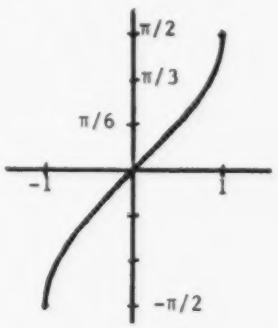
$$\sin^{-1}(\theta) \neq \frac{1}{\sin \theta}$$

2. Define inverse trigonometric functions.

2. If y = cos x then what is the inverse cosine function.

B. Graph the inverse trigonometric functions.

B. Graph $y = \sin^{-1} x$



C. Perform computations with inverse trigonometric functions.

C. If y = sin Arctan $[\frac{3}{2}]$ then what is value of y?

$$[y = \frac{3}{\sqrt{13}}]$$

D. Solve trigonometric equations.

D. Find all solutions if $0 \leqslant x \leqslant 2\pi$

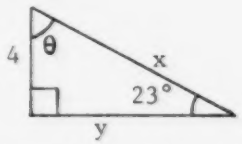
for $2 \sin x \cos x - \sin x = 0$

$[0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi]$

VI. Problem solving with triangles

A. Solve right triangles.

A. Given:



find all missing parts of the triangle
[x = 10.2, y = 9.4, θ = 67°]

B. Solve oblique triangles.

1. Derive and apply the law of sines.

1. Derive:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

2. Derive and apply the law of cosines.

2. Derive:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

VII. Vectors in the plane

A. Use notation and operations.

1. Describe vectors as ordered pairs and as directed line segments.

1. Describe: $\vec{u} = (4,1)$ as a directed line segment.

[the line segment joining (0,0) and (4,1)]

2. Define vector addition, scalar product and dot product.

2. Given $\vec{u} = (1,3)$ and $\vec{v} = (2,-1)$ then define: $\vec{u} + \vec{v}$, $2\vec{u}$, $\vec{u} \cdot \vec{v}$

[(3,2)], [(2,6)], [-1]

B. Use vectors in applications.

B. A fishing boat has a speed of 18 km/h. It wants to cross a river that has a current of 5 km/h. If the boat is to cross the river at right angles, what angle upstream should the boat be headed.

[16°]

VIII. Complex Numbers

A. Use definition and notation.

1. Define imaginary number.

1. Define: "i."

[$\sqrt{-1}$]

2. Introduce rectangular and polar notation for complex numbers.

2. Convert $\sqrt{3} + i$ to polar notation.

[$2 \text{ cis } \frac{\pi}{6}$]

B. Perform operations with complex numbers.

B. Find the product of 2 cis 30° and 6 cis 40°.

[12 cis 70]

C. Derive and use DeMoivre's Theorem to find powers and roots of complex numbers.

C. Note: This topic provides a good opportunity to discuss and illustrate mathematical induction.

Evaluate: $(1 + i)^{10}$

[32i]

ANALYTIC GEOMETRY

Introduction

Analytic geometry is outlined as a separate semester course. The course should include the topics in Sections I-VII. Additional optional topics to be studied if time permits include vectors, lines, planes, and surfaces in three dimensions.

The time allotted to the sections on vectors in the plane and polar coordinates may be shortened if these topics were covered well in trigonometry.

I. Vectors in the plane

- A. Vectors as ordered pairs
- B. Basic operations with vectors
- C. Relationships between vectors

II. Lines in the plane

- A. Vector equations for a line
- B. Cartesian equations for lines

III. Applications

- A. Distance between a point and a line
- B. Intersection of two lines
- C. Analytic proof of geometric theorems

IV. Conic sections

- A. Circles
- B. Parabolas, ellipses, and hyperbolas

V. Transformation of coordinates

- A. Translation of axes
- B. Completing of square
- C. Rotation of axes
- D. Discriminant method

VI. Curve sketching

- A. Polynomial functions
- B. Rational functions
- C. Parametric equations

VII. Polar coordinate system

- A. Polar coordinates
- B. Polar equations

VIII. Vectors in space

A. Cartesian coordinates in space
B. Basic operations with vectors
C. Cross product

IX. Lines and planes in space

A. Lines in space
B. Planes in space
C. Relationships between lines and planes
D. Distance between a point and a plane

X. Surfaces

A. Spheres
B. Cylinders and surfaces of revolution
C. Quadric surfaces

OBJECTIVES	EXAMPLES OR COMMENTS
The student should be able to:	
I. Vectors in the plane	
A. Use vectors as ordered pairs.	
1. Find the distance between two points.	1. Find the distance between $P_1(3,4)$ and $P_2(-4,8)$. [$\sqrt{65}$]
2. Describe a vector as a directed line segment.	2. Using vector notation, write the representation of the vector from $P_1(5,3)$ to $P_2(2,-1)$. [$\vec{v} = (-3,-4)$]
3. Determine a vector given its initial and terminal points.	3. Describe the vector $\vec{P_1 P_2} = ((1,2), (2,4))$ [a ray from (1,2) to (2,4)]
B. Use basic operations with vectors.	
1. Add and subtract vectors and illustrate graphically.	1. Let $\vec{v} = (-3,-1)$ and $\vec{u} = (2,-2)$. Then find $\vec{v} + \vec{u}$. [(-1,-3)]
2. Determine the norm (magnitude) and direction of a given vector.	2. Determine magnitude of $\vec{P_1 P_2}$ when $P_1(3,4)$, $P_2(2,-7)$. [$\sqrt{122}$]
3. Find the product of a real number and a vector.	3. Given: $P_1(3,4)$ $P_2(2,-7)$ $k = 6$, find value of $6\ \vec{P_1 P_2}$. [(-6,-66)]

4. Given a vector, find a unit vector in the same direction.

4. Let $\vec{v} = (2,3)$. Find a unit vector in the same direction.

$$\left[\left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)\right]$$

5. Find the inner product of two vectors.

5. If $\vec{u} = 6i + 4j + 2k$ and $\vec{v} = 2i - 2j - 6k$, then find the inner product of the two vectors.

[-8]

C. Show relationships between vectors.

1. Test for orthogonality using the inner product.

1. If $\vec{u} = 6i + 4j + 2k$ and $\vec{v} = 2i - 2j - 6k$, then are the two vectors orthogonal?

[no]

2. Determine the angle between two vectors.

2. Let $\vec{u} = (3,-2)$ and $\vec{v} = (-2, 5)$.

Determine the angle between the two vectors.

[2.43 radians]

3. Find the scalar projection of one vector onto another.

3. Find the scalar projection of $\vec{u}$ onto $\vec{v}$ above.

$$\left[\frac{-3\sqrt{2} - 10}{3\sqrt{3}}\right]$$

II. Lines in the plane

A. Write vector equations for a line.

1. Write parametric vector equations for the line determined by two points or a point and a direction.

1. Given the vector $\vec{v}$ from P(1,2) to Q(3,0) is $\overrightarrow{PQ} = (2,-2)$. Write the parametric vector equations for the line.

[x = 1 + 2t, y = 2 - 2t]

2. Determine if a given point lies on a given line.

2. Does P(3,4) lie on the line above?

[no]

3. Determine the slope of a line from a direction vector.	3. Find the slope of the line described in 1. [-1]
B. Recognize cartesian equations for lines.	
1. Recognize that $Ax + By + C = 0$ represents a line.	1. Rewrite $3 + 6y = 2x$ in general form for an equation of a line.
2. Find a cartesian equation for a line described in parametric form.	2. A point on the line represented by $x = 1 + 2t$ and $y = 2 - 2t$ for some real number t. Determine the equation of the line. [$x + y = 3$]
3. Determine if a pair of equations represents parallel or perpendicular lines.	3. Given: $3x + 4y = 2$ and $8y - 16 = -6x$. Determine whether the two lines are parallel or perpendicular. [parallel]
4. Write an equation for the line through two points.	4. Given: $P_1(2,3)$ and $P_2(4,12)$, find the equation of the line. [$2y - 9x + 12 = 0$]
5. Determine the slope and y-intercept of a line given its equation and conversely.	5. Given: $2x + 3y = 15$ find the slope and y-intercept. [$m = -\frac{2}{3}$, y-int. = 5]
III. Applications	
A. Find the distance between a point and a line.	A. Let the point $P(3,-1)$ and the line with equation $3x - y = 4$ be given. Find the distance from the point to the line. [$\frac{3}{5}\sqrt{10}$]

B. Find the point of intersection of two lines.

B. Find the point of intersection of $3x - 2y = 6$ and $-4x + 3y = 10$.

[38,54]

C. Prove some geometric theorems using cartesian coordinates and vectors.

C. Prove that diagonals of parallelogram bisect each other.

IV. Conic Sections

A. Use circle concepts.

1. Write a cartesian equation for a circle with given center and radius and conversely.

1. Find the center and radius of a circle whose equation is:

$x^2 + y^2 + 4x - 6y - 12 = 0.$

[center (-2,3) and radius 5]

2. Find an equation of the circle passing through three given points.

2. Find the equation of a circle passing through the points $P_1(3,3)$, $P_2(8,-2)$, and $P_3(7,1)$.

$[x^2 + y^2 - 6x + 4y - 12 = 0]$

B. Describe parabolas, ellipses, and hyperbolas.

1. Given an equation for a conic, find its geometric description, e.g., vertex, focus, center, etc.

1. Describe the graph of the equation:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

[an ellipse]

2. Given a geometric description of a conic, find its equation.

2. Find the equation of a parabola with focus (0,-2) and directrix $y = 2$.

$[x^2 = -8y]$

V. Transformation of coordinates

A. Translate axes.

A. A parabola has vertex V(3,4) and focus F(3,6). Find its equation.

$[(x - 3)^2 = 8(y - 4)]$

B. Complete square.

B. Identify the conic described by:

$x^2 - y^2 + 6x + 4y + 1 = 0.$

[hyperbola]

C. Rotate axes.

C. Determine the type of conic section given by:

$8x^2 - 4xy + 5y^2 - 36 = 0.$

[ellipse]

D. Discriminant method.

D. Use discriminant method to determine type of conic in C.

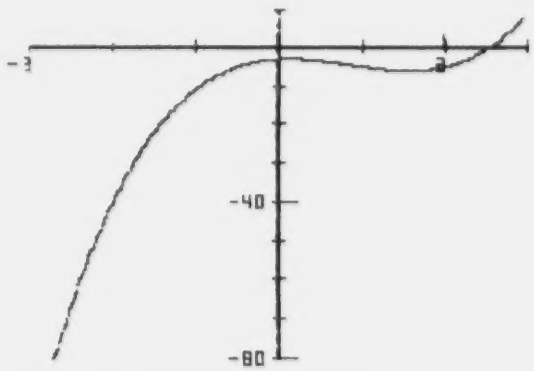
VI. Curve sketching

A. Describe polynomial functions.

1. Identify the shape of a graph of a given polynomial function of degree five or less and sketch its graph.

1. Graph the function:

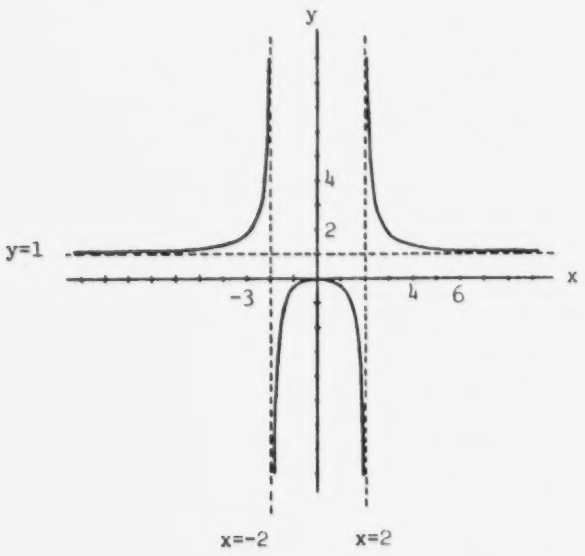
$y = 2x^3 - 5x^2 + x - 3$



B. Describe rational functions.

B. Graph the function:

$$y = f(x) = \frac{x^2}{x^2 - 4}$$

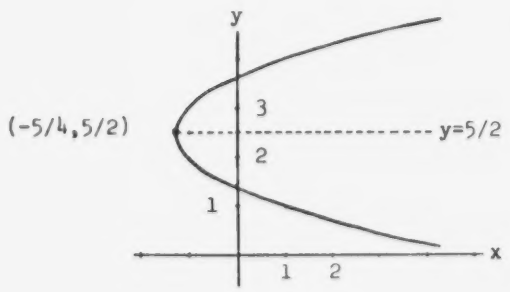


C. Describe parametric functions.

1. Sketch the graph of a curve described by parametric equations $x = f(t)$ and $y = g(t)$.

1. Sketch the graph of the following parametric equations:

$x = t^2 - 3t + 1$ and
$y = t + 1$



2. Find a cartesian equation from parametric equations for a curve.

2. Find the cartesian equation from the parametric equations in 1.

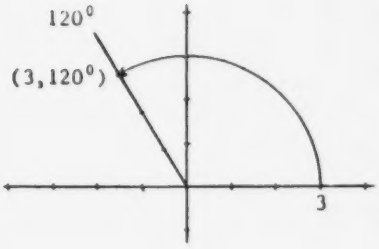
[$x = y^2 - 5y + 5$]

VII. Polar coordinate system

A. Graph with polar coordinates.

1. Graph a point with given polar coordinates.

1. Graph the point (3,120°).



2. Find alternate polar coordinate pairs for a given point.

2. Find alternate descriptions of (4,30°).

[(4,30° + k 360°), (-4,210° + k 360°) where k is any integer]

3. Convert between cartesian and polar coordinates for a point.

3. Convert (5,90°) to cartesian coordinates.

[0,5]

B. Use polar equations.

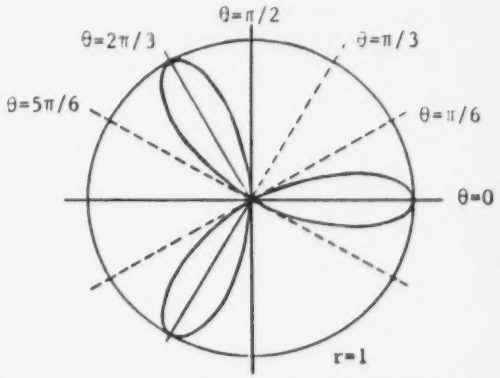
1. Transform between cartesian and polar equations.

1. Convert the equation $r = 4 \sin \theta$ to a cartesian equation.

[$x^2 + y^2 - 4y = 0$]

2. Graph polar equations representing lines, circles, roses, spirals, limacons, lemniscates, and conics.

2. Graph: $r = \cos 3\theta$ in polar coordinates.



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3. Determine the points of intersection of the graphs of two polar equations.

3. Given the equation of a circle $r = 2 \sin \theta$ and the equation of a cardioid $r = 2 - 2 \sin \theta$, find the point(s) of intersection.

$[(1, \frac{\pi}{6}), (1, \frac{5\pi}{6}), (0,0)]$

VIII. Vectors in space

A. Use cartesian coordinates in space.

1. Describe a right-hand coordinate system.

1. Define a right-handed coordinate system.

2. Find the distance between two points.

2. Find the distance between P(3,5,7) and Q(-2,4,-1).

$[3\sqrt{10}]$

B. Basic operations with vectors.

B. Let $\vec{u} = (2,-1,1)$ and $\vec{v} = (3,-2,2)$. What is the scalar projection of $\vec{u}$ onto $\vec{v}$?

$[\frac{10\sqrt{17}}{17}]$

C. Determine cross product.

1. Determine the cross product of two vectors and interpret geometrically.

1. Given: $\vec{u} = (2,-1,1)$ and $\vec{v} = (3,2,2)$, determine the cross product.

$[(-4,-1,7)]$

IX. Lines and planes in space

A. Describe lines in space.

1. Find a vector parametric equation for a line determined by a direction vector and a point or by two points.

1. Find the vector parametric equation for the line determined by P(2,2,-3) and Q(4,0,-1).

$[R = (4,0,-1) + t(2,-2,2)]$

2. Find the coordinates of the points that trisect a line segment.

2. Given: P(2,2,-3) and Q(4,0,-1), determine the points of trisection.

   [$(\frac{10}{3}, \frac{-2}{3}, \frac{-5}{3})$ and $(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3})$]

3. Determine symmetric equations for a line.

3. Given: P(2,3,4) and Q(3,-1,2) determine the symmetric equations for the line through P(2,3,4) and Q(3,-1,2).

   [$\frac{x-2}{1} = \frac{y-3}{-4} = \frac{z-4}{-2}$]

B. Describe properties of planes in space.

1. Given an equation of a plane, determine a normal vector to the plane and points on the plane.

1. Given: $4x + 2y + 8z = 0$ as the equation of a plane, determine a normal vector to the plane.

   [(4,2,8)]

2. Find an equation for the plane containing a point and having a given normal vector or containing three given points.

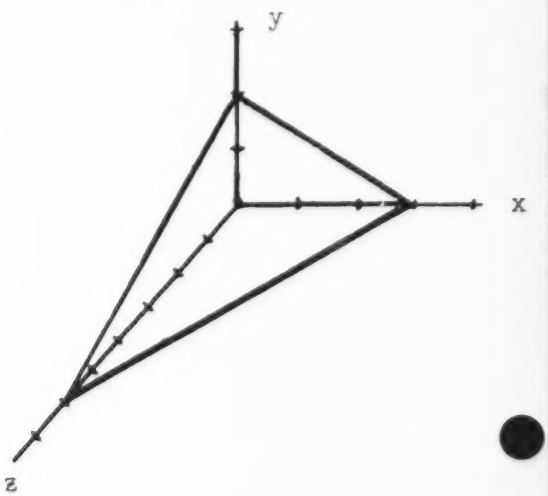
2. Given the plane P, with points (1,2,-1) (2,0,-1) (4,4,-3), determine the equation of the plane.

   [$4x + 2y + 8z = 0$]

3. Sketch the graph of a plane.

3. Graph: $2x + 3y + z = 6$.

   [the intercepts are: (3,0,0) (0,2,0) (0,0,6)]



C. Use relationships between lines and planes.

1. Determine if a given line is parallel or perpendicular to a plane.

1. Determine if the line L with equation $x = 1 + 2t$, $y = 3t$, $z = -2 + t$ and plane P with equation $2x - 3y + z = 5$ are parallel or perpendicular.

[neither]

2. Find an equation for the intersection of sets of planes.

2. Given: $3x + 2y + z = 12$ and $x + y + 2z = 3$. Find the equation of the intersection of the two planes.

[the intersection is a line with symmetric equations $\frac{x - 3}{3} = \frac{y - 2}{-5} = \frac{z - 3}{1}$]

D. Find the distance between a point and a plane.

D. Find the distance between the point (3,-1,2) and the plane $x + 2y - 2z = 6$.

[3]

X. Surfaces

A. Define spheres.

1. Find the equation of a sphere with given center and radius.

1. Determine the equation of a sphere with center (1,-2,3) and radius 4.

$[(x-1)^2+(y+2)^2+(z-3)^2 = 16]$

2. Find an equation of the plane tangent to a sphere.

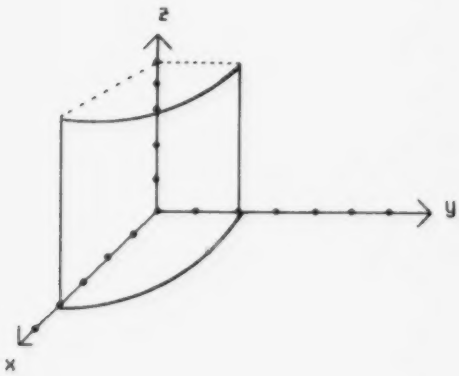
2. If P(x,y,z) is a point in the plane tangent to the sphere $x^2 + y^2 + z^2 = 9$, find the equation of the plane tangent to the sphere at the point T(1,-2,2).

[$x - 2y + 2z = 9$]

B. Discuss cylinders and surfaces of revolution.

1. Sketch the graphs of right cylinders in a cartesian coordinate system.

1. Sketch the first octant portion of the graph of the cylinder $x^2 + 4y^2 = 16$.



2. Find the equation of the surface formed by revolving a curve around one of the coordinate axes.

2. Determine the equation of the curve formed when the parabola $z = \sqrt{y}$ is revolved about the y axis.

[$y = x^2 + z^2$]

C. Determine quadric surfaces.

1. Using standard formulas, identify a given equation as representing an ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic cone, elliptic paraboloid, or hyperbolic paraboloid, and sketch the graph.

1. What quadric surface does the equation $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1$ represent?

[ellipsoid]

PROBABILITY AND STATISTICS

Introduction

A one semester course in probability and statistics will typically be broken into three sections. These are probability, descriptive statistics, and inferential statistics. Inferential statistics should be studied after some work in probability and descriptive statistics.

It is recommended that coins, cards, dice, and computers be used to simulate situations in the study of probability, and that calculators and computers be used to perform calculations throughout the course.

I. Probability

- A. Historical topics
- B. Definition of probability, sample space, and event
- C. Counting principles including permutations and combinations
- D. Odds
- E. Types of events including mutually exclusive, independent, and complementary
- F. Conditional probability and the use of tree diagrams
- G. Formulas for addition and multiplication of probabilities
- H. Probability distributions including the binomial distribution
- I. Mean and standard deviation of a probability distribution

II. Statistics

- A. Historical topics
- B. Organization of data including frequency distributions, histograms, bar graphs, and frequency curves
- C. Measures of central tendency including mean, median, and mode
- D. Measures of variation including variance and standard deviation
- E. Calculation of percentiles and z-scores
- F. Correlation and linear regression
- G. Discussion of improper and proper uses of statistics
- H. The standard normal distribution
- I. Standardization of a nonstandard normal distribution
- J. Normal approximation of a binomial distribution
- K. Sampling and the central limit theorem
- L. Estimation and the use of confidence intervals
- M. Decision making and the testing of hypotheses
- N. Activities in the actual collection and analysis of sample data

OBJECTIVES	EXAMPLES OR ACTIVITIES
The student should be able to:	
I. Discuss historical topics	
A. Appreciate the historical background for the development of probability as a branch of mathematics.	A. Individuals whose contributions can be used for classroom discussion include: Cardan, Descartes, Fermat, and Bernoulli. The teacher should consult one of the books on the history of mathematics in the related references.
B. Define probability, sample space, and event.	
1. Know the meaning of terms associated with basic probability: sample space, event, favorable and unfavorable outcomes.	1. A die is rolled. List the sample space. List the members of the event "the number showing is even." [1, 2, 3, 4, 5, 6] [2, 4, 6]
2. Know the definition of probability of an event and use it to find the probability of a described event.	2. A card is drawn from a standard deck. What is the probability it is a heart? $[\frac{1}{4}]$
C. Use counting principles including permutations and combinations.	
1. Find the number of outcomes by using the basic multiplication principle.	1. A sandwich consists of one of five different cheeses and one of four different breads. How many sandwiches are possible? [20]

2. Use the formulas for combinations and permutations to determine the number of outcomes in an event.

2. In how many ways can a committee of three be chosen from a group of seven?

   In how many ways can a bronze, silver, and gold medal be awarded in a seven person race?

   In how many ways can the letters of the word "statistics" be arranged?

   [$_7C_3 = 35$]

   [$_7P_3 = 210$]

   [$\frac{10!}{3!\ 3!\ 2!} = 50{,}400$]

D. Relate odds to events.

1. Relate the odds in favor of an event to the probability of the event occurring.

1. If Jimmy the Greek places odds in favor of the Minnesota Vikings winning the Super Bowl at seven to five, what does he feel is the probability of the Vikings winning?

   [$\frac{7}{12}$]

E. Identify types of events including mutually exclusive, independent, and complementary.

1. Find the probability of the complement of an event.

1. If the probability it will rain tomorrow is .3, what is the probability it won't rain?

   [.7]

2. Determine whether two stated events are mutually exclusive or independent.

2. A card is chosen from a deck. Are the events "a diamond is chosen" and "a queen is chosen" mutually exclusive? Are they independent?

   [no, yes]

F. Use conditional probability and the use of tree diagrams.

1. Use a tree diagram to determine the outcomes in a sample space.

1. A family has two children. Find the sample space for the sex of the children.

[
]

2. Find probabilities by using the formula for conditional probability.

2. $p(A) = .3$,
$p(B) = .2$,
$p(A \cap B) = .1$.
Find $p(A/B)$.

[.5]

G. Use formulas for addition and multiplication of probabilities.

1. Use formulas for addition and multiplication of probabilities to find probabilities for union and intersection of events.

1. $p(A) = .3$,
$p(B) = .2$,
$p(A \cap B) = .06$.
Find $p(A \cup B)$.
Are A and B independent?

[.44, yes]

H. Chart probability distributions including the binomial distribution.

1. Chart the complete probability distribution for a described variable.

1. If x represents the number of boys in a family with three children, find the complete probability distribution for x.

[

0	1	2	3
1/8	3/8	3/8	1/8

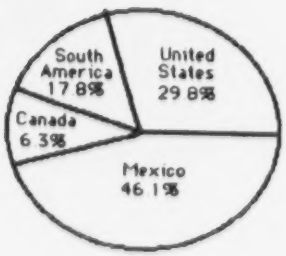
]

2. Find the probability for a given value of the random variable associated with repeated independent trials of a binomial random variable.	2. If a coin is tossed ten times, find the probability of obtaining exactly five heads. $[{}_{10}C_5(.5)^5(.5)^5 \cong .2461]$
I. Find mean and standard deviation of a probability distribution.	
1. Find the mean and standard deviation for a random variable.	1. Find the mean and standard deviation for the random variable in the first example under H. $\left[\begin{array}{l}\text{mean} = 1.5 \\ \text{standard deviation} = \frac{\sqrt{3}}{2}\end{array}\right]$
2. Find the mean and standard deviation for the random variable associated with repeated independent trials of a binomial random variable.	2. Find the mean and standard deviation for the random variable in the second example under H. $\left[\begin{array}{l}\text{mean} = (10) \cdot (.5) = 5 \\ \text{standard deviation} = \\ \sqrt{10 \cdot .5 \cdot .5} = \sqrt{2.5}\end{array}\right]$
II. Statistics	
A. Discuss historical topics.	
1. Appreciate the historical background for the development of statistics as a branch of mathematics.	1. The teacher should consult one of the books on the history of mathematics in the related references.

B. Organize data including frequency distributions, histograms, bar graphs, and frequency curves.

1. Read and interpret histograms, circle graphs, picture graphs, and frequency curves.

1.



The chart shows the estimated distribution of oil in the Western Hemisphere. If there are 150 billion barrels of oil in the Western Hemisphere, how many billion barrels does the U.S. have?

[44.6]

2. Organize data into a frequency table by individual items or groups of items and represent the data in a histogram, circle graph, or frequency curve.

2. Collect data on the heights of the students in class and members of their families. Arrange the heights into ten groups, construct a frequency distribution showing the classes, class marks (midpoints), and the number in each class. Draw a histogram and frequency curve. Draw a circle graph showing the portion over and under 5'6".

C. Calculate measures of central tendency including mean, median, and mode.

1. Find the mean, median, and mode for a set of data.

1. Find the mean, median, and mode for the data collected in the height of students previous activity.

D. Calculate measures of variation including variance and standard deviation.

1. Find the range, variance, and standard deviation for a set of data.

1. Find the range, variance, and standard deviation for the set of data, on the heights of students.

E. Calculate percentiles and z-scores

1. Calculate the percentile rank and z-score for an individual item in a set of data.

1. Find a percentile rank and z-score for a selected height in the set of data used in D.

F. Calculate correlation and linear regression.

1. Construct a scatter diagram for paired data and estimate correlation.

1.a. What type of correlation would you expect between the age of a car and its value?

[negative]

1.b. Collect the height and shoe size (add 1½ to a girl's shoe size) of each class member and construct a scatter diagram. Inspect for correlation.

2. Calculate the correlation coefficient for paired data.

2. Calculate the correlation coefficient for the data in the previous activity. (The use of a computer is appropriate here. It is not useful to expect the students to do this without a computer for a set of data with more than a few pairs.)

3. Find the linear regression equation and use it to make a prediction.

3. Find the linear regression equation for the data in the previous activity and use it to predict a height/shoe size from a selected shoe size/height.

G. Discuss improper and proper uses of statistics.

1. Criticize statistics found in the real world.

1. Examples for class discussion can be found in books on statistics listed in the related references. Graphs can be made into transparencies for class discussion or copied and distributed to students for criticism. Besides graphs, poorly used averages and the mistaken notion that correlation implies causation are particularly worth discussing.

H. Determine the standard normal distribution.

1. Use a table to determine the probability that the standard normal variable, z, will fall in a particular region.

1.a. Use the standard normal tables to find the following probabilities:

$p(z > .87)$

$p(z < 1)$

$p(-1.63 < z < 2.22)$

[.1922, .8413, .9352]

1.b. The probability of getting less than a certain z-value is .1190. Find the z-value.

[-1.18]

I. Understand standardization of a nonstandard normal distribution.

1. Write an equation to standardize a normally distributed variable and use the equation to determine probabilities for the normal variable.

1. IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. If x represents an unknown IQ score and z is the standard normal random variable, write an equation to standardize x and find the probability $x > 110$.

$$\left[z = \frac{x - 100}{15} \right]$$

[.2514]

J. Determine normal approximation of a binomial distribution.

1. Use the standardizing procedure to determine approximate probabilities for repeated independent trials of binomial random variable.

1. If a coin is tossed 100 times, find the probability of obtaining less than 40 heads.

[.0228]

K. Determine sampling and use the central limit theorem.

1. Use random numbers to obtain a random sample from a group.

1. The instructor might use a table, computer, or calculator to select three or four students to write their solutions to assigned problems on the chalkboard.

L. Estimate and use confidence intervals.

1. Apply the central limit theorem to questions and standard deviation.

1.a. An experiment to demonstrate the conclusions of the central limit theorem might consist of dividing the class into groups of two or three students and providing each group with a bag and 20 plastic or wooden chips that include: five marked with a "3," four with a "6," two with a "9," four with a "12," and five with a "15." Each group should be asked to make 50 draws of three chips each and record the mean of each draw. Results from all groups can be pooled into one large distribution of sample means. A histogram can be drawn and the mean and standard deviation calculated for both the population and the distribution of sample means. Symbols such as:
$\bar{x}, \mu, \sigma, \mu_{\bar{x}}, \sigma_{\bar{x}}$,
should also be discussed.

1.b. The length of a ruler produced by the Meter Company is 25 cm with a standard deviation of .31 cm. If two dozen rulers are selected at random, what is the probability the average mean is more than 25.1 cm?

[.0571]

2. Calculate a statistical confidence interval for a correlation mean.

2. A sample of 100 families in Fargo found that a family of four spent an average of $46 a week for food with a standard deviation of 5.6. Find a 95 percent confidence interval for the average amount spent on food by a family of four in Fargo.

   [44.90, 47.10]

3. Use the central limit theorem to determine sample size.

3. A medical researcher experimenting with a new drug wants to estimate the average recovery time of patients treated with this drug. How large a sample should be taken if the researcher wants to be within two hours of the true value? Assume σ = 7.39 and a 95 percent confidence level.

   [53]

M. Demonstrate decision making and the testing of hypotheses.

1. Use the central limit theorem to decide whether to accept or reject a particular hypothesis at a particular level of significance.

1.a. Terms such as null hypothesis, significance level, test statistic, type I and II errors should be discussed.

1.b. The manufacturer of a movie projector bulb claims an average life of 25 hours with a standard deviation of 1.69 hours. A survey of 50 bulbs showed an average life of 23 hours. Should we accept or reject the manufacturer's claim at the 5 percent level of significance?

   [test statistic is -8.37, therefore reject claim]

N. Perform the actual collection and analysis of sample data.

1. Collect data from the real world and use statistics to analyze the data.

1. An excellent way to end the study of inferential statistics is an actual experience in the drawing of conclusions by means of a random sample. High school students enjoy polling their fellow students on issues of current importance. A poll using a randomly selected portion of the student body or community can be followed by the analysis of the data through the calculation of confidence intervals, correlation coefficients, etc.

MATHEMATICAL APPLICATIONS

I. Problem solving

A. Understanding the problem
B. Devising a plan
C. Carrying out the plan
D. Looking back

II. Informal geometry

A. Basic concepts

1. Points, lines, planes, space
2. Line segments, rays, angles
3. Measures of segments and angles
4. Constructions

B. Angles

1. Classification by measure
2. Angles formed by a transversal cutting parallel lines
3. Angles formed by perpendicular lines
4. Complementary and supplementary angles
5. Constructions

C. Triangles

1. 180° in a triangle
2. Altitudes, medians, angle bisectors
3. Special triangles
4. Congruence and similarity
5. Ratio and proportion
6. Right triangle trigonometry
7. Constructions

D. Polygons

1. Quadrilaterals
2. Pentagons, hexagons, octagons

E. Circles

1. Radius, diameter, chord
2. Central angles
3. Tangents

F. Measures

1. Use of formulas
2. Perimeters and circumferences
3. Area
4. Volume
5. Surface area

III. Everyday statistics

A. Exploring data

1. Reading and interpreting graphs
2. Organizing data into tables and graphs
3. Measures of central tendency
4. Measures of variation

B. Probability

1. Probability of simple events
2. Probability of compound events
3. Probability distributions

C. Drawing conclusions from data

1. Sampling
2. Correlation
3. Estimation

IV. Mathematics of finance

A. Earning money

1. Payroll

a. Wages
b. Taxes

2. Investments

a. Savings
b. Real estate

B. Managing money

1. Banking
2. Loans

C. Spending money

1. Pricing
2. Purchasing

OBJECTIVES

The student should be able to:

I. Problem solving

A. Understand the problem.

1. Determine the unknown(s).
2. Restate data.
3. Draw a diagram.
4. Make a chart or table.
5. Determine if sufficient information is given.

B. Devise a plan.

1. Find the connection between the data and the unknown.
2. Use resources to gather necessary information.
3. Relate to familiar problems.
4. Examine special cases.
5. Generalize the problem.
6. Simplify the problem.
7. Estimate the solution.

C. Carry out the plan.

D. Look back.

1. Check the solution.
2. Review the strategy.
3. Relate to other problems.

II. Informal geometry

A. Demonstrate basic concepts.

1. Demonstrate the undefined concepts of points, lines, planes, and space.

2. Demonstrate the concepts of line segments, rays, and angles.

3. Use rulers and protractors.

4. Construct equal angles, line segments, and bisectors of angles and line segments.

B. Identify angles.

1. Recognize acute, right, and obtuse angles.

2. Identify vertical, adjacent, corresponding, and alternate interior angles.

3. Recognize perpendicular lines.

4. Identify complementary and supplementary angles.

5. Construct parallel and perpendicular lines, complementary and supplementary angles, etc.

C. Use triangle properties.

1. Demonstrate that the sum of the angles of any triangle is 180°.

2. Identify altitudes, medians, and angle bisectors of triangles--discuss center of gravity, inscribed circles, and circumscribed circles.

3. Use the relationships in isosceles, equilateral, 30°-60°-90°, and 45°-45°-90° triangles.

4. Determine congruent triangles by tracing and discuss correspondence between parts. Recognize similarity between triangles.

5. Work with ratio and proportion to find segment lengths of similar triangles.

6. Solve problems using sine, cosine, and tangent ratios.

7. Construct altitudes, medians, and angle bisectors of triangles. Construct congruent triangles.

D. Use polygon properties.

1. Identify various types of common quadrilaterals and demonstrate special characteristics of each.

2. Recognize pentagons, hexagons, and octagons.

E. Demonstrate circle properties.

1. Identify the parts of a circle, i.e. radius, diameter, chords, arcs, sectors, segments.

2. Demonstrate the relationship between central angle and intercepted arcs.

3. Recognize a tangent to a circle.

F. Use formulas to measure varies quantities.

1. Substitute numbers into formulas correctly.

2. Determine a formula and use it to calculate perimeters and circumferences.

3. Determine a formula and use it to calculate areas.

4. Determine a formula and use it to calculate volume.

5. Determine a formula and use it to calculate surface area.

III. Everyday statistics

A. Exploring data

1. Read and interpret graphs.

a. Read and interpret histograms, circle graphs, picture-graphs, and frequency polygons.

b. Recognize errors commonly made in the labeling and scaling of graphs.

2. Organize data into tables and graphs.

a. Organize data into a frequency table.

b. Organize data into graphs including histograms, circle graphs, frequency polygons, stem and leaf plots, and scatter diagrams.

3. Measure central tendency.

a. Calculate the mean, median, and mode of a set of data.

b. Select an appropriate average to represent a set of data.

4. Calculate variation.

a. Calculate the percentile rank of an item.

b. Calculate the range and average deviation of a set of data.

B. Probability

1. Calculate probability of simple events.

   a. Find the probability of a simple event by using the definition of probability.

   b. Find the approximate probability of a simple event by experimentation.

   c. Determine whether two events are mutually exclusive.

   d. Determine whether two events are independent.

2. Find probabilities of compound events.

   a. Find probabilities of compound events by using tree diagrams or experimentation.

   b. Find the probability of the union of two mutually exclusive events by addition.

   c. Find the probability of the intersection of two independent events by multiplication.

3. Construct probability distributions.

   a. Construct a table and bar graph showing the probabilities for an experiment with several outcomes.

   b. Estimate probabilities from a scaled normal curve.

C. Draw conclusions from data.

1. Understand sampling.

   a. Select a sample from a group and check for randomness.

   b. Use random digits or simulation methods to select a sample from a group.

2. Understand correlation.

   a. Determine if verbally described paired data would have positive, negative, or no correlation.

   b. Estimate the correlation from a scatter diagram by comparing to scatter diagrams with known correlation. (Calculation of the actual value of the correlation coefficient is recommended where a computer or suitable calculator is available.)

   c. Understand that a cause and effect relationship may not accompany correlation.

3. Estimate using statistics.

a. Predict the value of one variable from a given value of another using a scatter diagram.

b. Calculate the sample proportion for a random sample from a yes-no population and estimate the population proportion.

c. Find the endpoints of a confidence interval for the population proportion by using the sample proportion and the margin of error.

IV. Mathematics of finance

A. Earning money

1. Calculate payrolls.

a. Calculate wages using time cards (regular and overtime).

b. Compute withholding taxes and personal deductions.

2. Discuss investments.

a. Calculate simple and compound interest on savings accounts using calculators and tables.

b. Calculate rate of return on items of investment bought at discount, such as treasury notes or bonds.

c. Calculate net return and rate of return on real estate investments.

B. Managing money

1. Understanding banking concepts.

a. Write checks and maintain a check record.

b. Reconcile a checking account, using information from a bank statement and a check record.

2. Compute loan charges.

a. Compute the maturity value and interest rate of a single payment loan.

b. Compute the amount financed and the finance charge on an installment loan.

c. Use a table to find the annual percentage rate of a loan.

d. Compute the mortgage payments and analyze the repayment terms under varying parameters.

C. Spending money

1. Understand pricing structures.

   a. Use a calculator to find and compare unit prices of items.

   b. Compute the markup in dollars and as a percent of the selling price.

2. Understand purchasing procedures.

   a. Calculate discounts when given the rate of discounts.

   b. Calculate rate of discount when given the discount.

   c. Calculate amount of saving knowing sale price and rate of discount.

   d. Use tables to calculate insurance premiums.

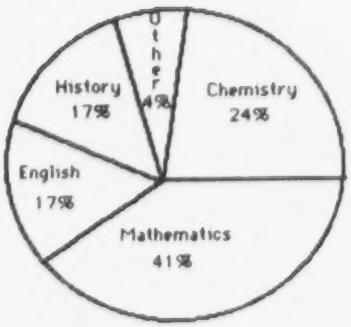
Sample Problems

Students should be encouraged to use the previously described problem solving strategy. The variety of problems encountered is large, but the strategy applies to all problems. Its consistent use will improve both qualitative and quantitative reasoning. Three typical problems follow.

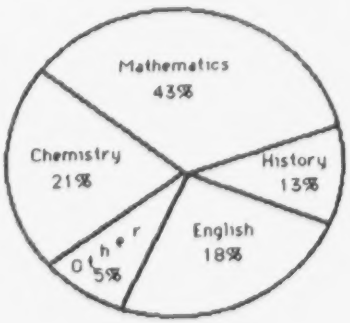
Sample 1.

MAIN ACADEMIC INTEREST AT BISMARCK JUNIOR COLLEGE

MEN



WOMEN



Women comprise 55 percent of the 2,500 students at Bismarck Junior College. For the statements that follow, select:

1 - if the statement is totally supported by the given information,
2 - if the statement is contradicted by the given information,
3 - if the information is insufficient to decide.

A. More women than men have main interest in chemistry.

1 2 3 Answer: 1

B. More sophomore women have main interest in history than freshmen men with main interest in chemistry.

1 2 3 Answer: 3

C. Fewer women than men have main interest in history.

1 2 3 Answer: 2

Sample 2.

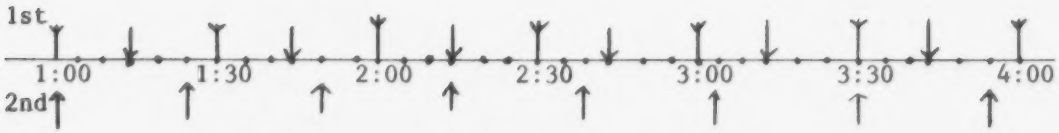
A mixture consists of eight parts sand and five parts cement. How many kilograms of cement are there in 260 kilograms of the mixture?

Solution: The unknown is the number of kilograms of cement, so let this be denoted by k. The ratio of sand to cement is 8 to 5, and there are 260 - k kilograms of sand. Hence 8 is to 5 as 260 - k is to k. In equation form we have $8/5 = (260 - k)/k$.

This can be changed to $8k = 1{,}300 - 5k$, so $13k = 1{,}300$ and finally we arrive at $k = 100$. There are 100 kilograms of cement in 260 kilograms of the mixture.

Sample 3.

One new style of watch is programmed to beep at 15 minute intervals and another beeps at 25 minute intervals. If they beep together at 1:00 p.m., what is the first time after midnight at which they again will beep together?



Solution: From examining the above table, it appears that they will beep together every 75 minutes. There are 660 minutes between 1:00 p.m. and midnight. $600/75 = 8.8$ which means they beep together 8 times prior to midnight and are 8/10 of the way toward beeping together again. 75 times .8 is 60; therefore, 15 minutes more must lapse following midnight before they beep together again. The answer is 12:15 a.m.

APPENDIX A

CALCULATORS IN MATHEMATICS EDUCATION

The widespread availability and use of calculators cannot be ignored when developing a State Curriculum Guide. Calculators are now accepted as efficient tools for computation. Because of their use and place in society and their value in teaching and learning mathematics, calculators must be incorporated into the school mathematics program.

The National Council of Teachers of Mathematics continues to endorse the minicalculator as a valuable instructional aid for mathematics education and to recommend the use of the minicalculator in the classroom. NCTM has published the following position statement: "With the decrease in the cost of the minicalculator, its accessibility to students at all levels is increasing rapidly. Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom, the minicalculator should be used in imaginative ways to reinforce learning and to motivate the learner as he/she becomes proficient in mathematics."

The Council's Instructional Affairs Committee identified nine ways in which the minicalculator can be used in classrooms:

1. To encourage students to be inquisitive and creative as they experiment with mathematical ideas.

2. To assist the individual to become a wiser consumer.

3. To reinforce the learning of the basic number facts and properties in addition, subtraction, multiplication, and division.

4. To develop the understanding of computational algorithms by repeated operations.

5. To serve as a flexible "answer key" to verify the results of computation.

6. To promote student independence in problem solving.

7. To solve problems that previously have been too time-consuming or impractical to be done with paper and pencil.

8. To formulate generalizations from patterns of numbers that are displayed.

9. To decrease the time needed to solve difficult computations.

In the National Council of Teachers of Mathematics an Agenda for Action: Recommendations for School Mathematics of the 1980s, we find the following actions:

3.1 All students should have access to calculators and increasingly to computers throughout their school mathematics program.

3.2 The use of electronic tools such as calculators and computers should be integrated into the core mathematics curriculum. Calculators should be available for appropriate use in all mathematics classrooms, and instructional objectives should include the ability to determine sensible and appropriate uses. Calculators and computers should be used in imaginative ways for exploring, discovering, and developing mathematical concepts and not merely for checking computational values or for drill and practice.

From the primary grades on, calculators should be used for exploratory activities. As students understand basic concepts and learn the basic arithmetic operations, they should be taught how and when to use the calculator. As more experience is gained, effective use of calculators should greatly increase. As this happens, it is especially important to emphasize understanding of place value and to emphasize that the results of the calculation must be reasonable.

In the upper elementary grades, students should be performing computations with calculators on a regular basis. At the same time, they should learn that for some simple computations, use of the calculator is cumbersome and can obscure the understanding of the concepts. As they gain experience, students should be expected to judge whether use of the calculator will be effective and efficient. Before the end of the sixth grade, students should have calculators continuously available for use in class, on homework assignments, and on tests. By the time students move into secondary schools, the use of calculators should be routine.

Calculators cannot serve as a substitute for good teaching, nor for use of concrete manipulatives in the classroom. They should not be used for "busy work" drill and practice. Most important, the use of calculators must not replace the development of the students' understanding of the meaning of arithmetic operations and the common algorithms used to perform those computations. With full and appropriate use of the calculator, all students can have the time and instructional support to learn to think through problems, knowing that lengthy computations will not be a barrier to success.

Classroom sets of calculators should be available for student use in all schools and their use integrated into the mathematics curriculum.

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Mauland, Lyle E., and Prigge, Glenn R. Activities for the Minicalculator. North Dakota Council of Teachers of Mathematics, Minot, North Dakota, 1977.

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Ockenga, Earl, and Immerzeel, George. Calculator Activities (Book 1). Creative Publications, Inc., Palo Alto, California.

Padberg, Friedhelm F. "Using Calculators to Discover Simple Theorems - An Example From Number Theory." Arithmetic Teacher. 1981 (April), 28:21-23.

Prigge, G. R., and Gawronski, J. D. Calculator Activities. Math-Master, Big Spring, Texas, 1978.

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VanDevender, Evelyn M., and Rice, Dale R. "Improving Instruction in Elementary Mathematics With Calculators." School Science and Mathematics. 1984 (December), 84:633-643.

Weaver, J. F. "Calculators and Unary Operations." School Science and Mathematics. 1981 (March), 81:567-573.

Wiebe, James H. "Using a Calculator to Develop Mathematical Understanding." Arithmetic Teacher. 1981 (November), 29:36-38.

Woodward, Ernest. "Calculators With a Constant Arithmetic Feature." Arithmetic Teacher. 1981 (October), 29:40-41.

APPENDIX B

COMPUTERS IN MATHEMATICS EDUCATION

With computers now readily available in most schools, and with the increasing availability of good software, it is clear that computers will become major tools of education. With the continuing efforts by the computer industry to provide "user friendly" software, schools no longer need an expert programmer to make use of current computer technology. The problem becomes one of proper utilization of this technology. The best role of computers in the educational process must be assessed by each school. The purpose of this section is to attempt to provide an overview of various activities involving computers that may be useful in a mathematics curriculum. The text of this section is followed by a listing of reviewers of educational computer software and guides available for help in reviewing software. Also included is a "Request for Information Form Letter," a "Software Evaluation Checklist" and a "Software Documentation Sheet."

The computer can assist instruction in three ways. First, software is available to supplement "Drill and Practice" exercises. "Drill and Practice" can be defined as those exercises used to provide routine practice of an objective or set of objectives. They are designed to reinforce and/or remediate previously learned skills and knowledges. These software programs are electronic versions of worksheets and are designed to provide immediate feedback to students. Many of the drill and practice programs provide various levels of drill as well as a student management component which records student performance and make reports available to the teacher.

Drill and practice programs can be used to reinforce concepts at both the elementary and secondary level. These programs provide the student with additional methods of practicing and reinforcing basic skills. Programs are available to supplement traditional elementary classroom activities such as those dealing with fractions, speed drills, arithmetic operations, money, statistics, and geometry. At the secondary level, activities may include solving of binomial expressions, linear equations, quadratic equations, statistical analysis and trigonometric calculations in addition to graphing functions. These programs are not meant to supplant, but rather to supplement teacher-student interaction. These computer exercises can be done individually or in small groups.

The computer may be used as a tutorial aid. This can be accomplished either through Computer Assisted Instruction (CAI) packages or commercially prepared software. The CAI programs can often be time consuming for the teacher to prepare properly and necessitate the clear determination of the objectives to be taught. Since these programs originate in the school, they will address specific school objectives. Commercially prepared tutorial software may be more expensive and may not address the specific objectives desired. Commercial software is usually more "professionally" presented and therefore may present the vendor's objectives more completely and clearly.

Tutorial programs can be very useful for individualized instruction. A properly chosen program can be useful for enriching the talented student, or remediating the slower student. Programs can be used to introduce topics that are not normally covered in a particular course or grade level. For example, a program can be used a) to introduce multiplication to an advanced second grade student; b) to give additional help to a fourth grade student having difficulty with transposing in subtraction; c) to aid an eighth grade student who is having trouble with long division; d) to introduce an advanced eighth grade student to beginning algebra; e) to assist a twelfth grade student who has not taken math for two years, to "brush up" on the metric system or basic algebra.

Finally, the computer can be used to enhance problem solving techniques through the use of simulations, spreadsheets, and programming. Simulations create a variety of opportunities for stimulating student involvement in problem solving. Simulations are game-like activities that allow students to make decisions, and interact in situations that are often too complex, historical, expensive, or distant to bring into the classroom. They are computer-based models of real-life situations which allow students to control certain variables and provide ways of checking their hypothesis about those variables. Programs may or may not be directly related to math, but their end results, i.e., hypothesis-forming and testing, are consistent with skills addressed in most math curricula.

A second method of developing problem solving techniques is through the use of spreadsheets. Spreadsheets allow the student to input data, perform calculations on the data, and to organize the results in a meaningful way. For example, students may input various interest rates for paying back a loan, calculate the monthly payments, and finally organize the data to relate interest rate with monthly payment. The spreadsheet can also be used to perform various calculations with a function and then list the results, and finally organize the results to see relationships between the variables and function. This type of exercise allows the student to focus specifically on relationships without getting involved in tedius time consuming calculations.

A final method to improve the problem solving abilities of students is to have them create their own programs. This does not necessitate developing an extensive ability to program, but rather may simply involve the utilization of a limited number of commands of one language such as BASIC or LOGO. These commands are then used to perform specific tasks such as calculating the monthly balance on a car loan or forming a geometric figure. Programming forces the student to define the problem, formulate, implement, and evaluate a plan. This can be accomplished in a nonthreatening environment, with immediate feedback and reinforcement while challenging the student's creativity.

In teaching mathematics, as with any subject, educators must be aware of individual learning styles. The computer is one tool that can be used to facilitate the learning of mathematics. It should be used to extend and enrich the learning experiences available to students.

Reviewers of Computer Software

EPIE (Educational Products Information Exchange). EPIE Institute, Box 620, Stony Brook, NY 11790.

MICROSIFT (Microcomputer Software Information for Teachers). Computer Technology Program, Northwest Regional Educational Laboratory, 300 Southwest Sixth Avenue, Portland, OR 97204.

Minnesota Educational Computing Consortium (MECC). MECC, Instructional Systems Development, 2520 Broadway Drive, St. Paul, MN 55113.

School Microware: A Directory of Educational Software. Dresden Associates, P.O. Box 246, Dresden, ME 04342.

Additional Guides for Evaluating Software

AEDS MONITOR 20 (April, May, June, 1982).

Microcomputer Software for Instructional Use: Where are the Critical Reviews?" by Ann Lathrop, The Computing Teacher 9 (February, 1982): 22-26.

National Council of Teachers of Mathematics (NCTM). NCTM, 1906 Association Drive, Reston, VA 22091. $3.75.

SAMPLE REQUEST-FOR-INFORMATION FORM LETTER

_____
_____
_____

Dear _____:

We wish to evaluate some of your computer programs for possible purchase. This evaluation would include any related teacher- or student-support materials. The following programs are of specific interest:

_____ _____
_____ _____
_____ _____

Before requesting the programs themselves, we need information (brochures, technical reports, or your comments) to answer the following questions:

1. What are the system and memory requirements for each program's use?
2. On what media (disks, cassette tape, cartridge, etc.) is each program available?
3. In what computer language(s) is each program written?
4. For which grade levels or ability levels is each program intended?
5. What are the specific content areas or instructional objectives for each program?
6. Has each program been tested or validated in a classroom setting?
7. What kind of instructional materials are available to support a given program's use? What is the cost of such materials?
8. What is the procedure or cost for obtaining each program for a full review?
9. What is the purchase price of each program?

Please send this information to:

_____
_____
_____

Thank you for your assistance.

Sincerely,

_____
_____

SMC: 3/86

SOFTWARE EVALUATION CHECKLIST

PROGRAM NAME: _____ SOURCE: _____ COST: _____

SUBJECT AREA: _____ REVIEWER'S NAME: _____ DATE: _____

SMC: 3/86

1. INSTRUCTIONAL RANGE

_____ grade level(s)

_____ ability level(s)

2. INSTRUCTIONAL GROUPING FOR PROGRAM USE

_____ individual

_____ small group (size: _____)

_____ large group (size: _____)

3. EXECUTION TIME

_____ minutes (estimated) for average use

4. PROGRAM USE(S)

_____ drill or practice

_____ tutorial

_____ simulation

_____ instructional gaming

_____ problem solving

_____ informational

_____ other (_____)

5. USER ORIENTATION: INSTRUCTOR'S POINT OF VIEW

low				*high*	
•	•	•	•	•	flexibility
•	•	•	•	•	freedom from need to intervene or assist

6. USER ORIENTATION: STUDENT'S POINT OF VIEW

low				*high*	
•	•	•	•	•	quality of directions (clarity)
•	•	•	•	•	quality of output (content and tone)
•	•	•	•	•	quality of screen formatting
•	•	•	•	•	freedom from need for external information
•	•	•	•	•	freedom from disruption by system errors
•	•	•	•	•	simplicity of user input

7. CONTENT

low				*high*	
•	•	•	•	•	instructional focus
•	•	•	•	•	instructional significance
•	•	•	•	•	soundness or validity
•	•	•	•	•	compatibility with other materials used

8. MOTIVATION AND INSTRUCTIONAL STYLE

passive				*active*	
•	•	•	•	•	type of student involvement

low				*high*	
•	•	•	•	•	degree of student control

none	*poor*			*good*	
•	•	•	•	•	use of game format
•	•	•	•	•	use of still graphics
•	•	•	•	•	use of animation
•	•	•	•	•	use of color
•	•	•	•	•	use of voice input and output
•	•	•	•	•	use of nonvoice audio
•	•	•	•	•	use of light pen
•	•	•	•	•	use of ancillary materials
•	•	•	•	•	use of _____

9. SOCIAL CHARACTERISTICS

present and negative	*not present*	*present and positive*	
_____	_____	_____	competition
_____	_____	_____	cooperation
_____	_____	_____	humanizing of computer
_____	_____	_____	moral issues or value judgments
_____	_____	_____	summary of student performance

B-5

DIRECTIONS FOR SOFTWARE EVALUATION CHECKLIST

1. The grade levels and ability levels for a particular program are primarily determined by the concepts involved. Other important factors are reading level, prerequisite skills, degree of student control, and intended instructional use. It is possible for a program to be flexible enough to be used across a wide range of grade levels and ability levels.

2. Some programs are designed for use by individuals. Others have been or can be modified for participation by two or three persons at a time. Simulations or demonstrations often pose opportunities for large-group interaction. A given program may be used in more than one grouping, depending on the instructor.

3. The time required for the use of a program will vary considerably. Include loading time for cassettes. A time range is the appropriate response here.

4. Instructional programs can be categorized according to their uses. Some programs may have more than one use, thus falling into more than one of the following categories:
   Drill or practice: Assumes that the concept or skill has been taught previously.
   Tutorial: Directs the full cycle of the instructional process; a dialogue between the student and the computer.
   Simulation: Models selected, alterable aspects of an environment.
   Instructional gaming: Involves random events and the pursuit of a winning strategy.
   Problem solving: Uses general algorithms common to one or more problems.
   Informational: Generates information (data).

5. These are factors relevant to the actual use of the program from the point of view of an instructor.
   Flexibility: A program may allow the user or the instructor to adjust the program to different ability levels, degrees of difficulty, or concepts.

   Intervention or assistance: A rating of "low" means considerable teacher intervention or assistance is required.

6. These are factors relevant to the actual use of the program from the point of view of a student.
   Directions: The directions should be complete, readable, under the user's control (e.g., should not scroll off the screen until understood), and use appropriate examples.

   Output: Program responses should be readable, understandable, and complete. If in response to student input, the output should be of an acceptable tone and consistent with the input request.

   Screen formatting: The formats during a program run should not be distracting or cluttered. Labels and symbols should be meaningful within the given context.

   External information: A program may require the user to have access to information other than that provided within it. This may include prerequisite content knowledge or knowledge of conventions used by the program designer as well as maps, books, models, and so on.

   System errors: System errors result in the involuntary termination of the program.

   Input: A program should ensure that a user knows when and in what form input is needed. It should avoid using characters with special meanings, restrict input locations to particular screen areas, and require minimal typing.

7. These are matters relevant to the subject-matter content of the program.
   Focus: The program topic should be clearly defined and of a scope that permits thorough treatment.

   Significance: The instructional objectives of the program must be viewed as important by the instructor. Also, the program should represent a valid use of the computer's capabilities while improving the instructional process.

   Soundness or validity: The concepts and terms employed should be correct, clear, and precise. Other important factors are the rate of presentation, degree of difficulty, and internal consistency.

   Compatibility: The content, terminology, teaching style, and educational philosophy of the program should be consistent with those generally encountered by the student.

9. Competition, cooperation, and values are concerns that may be a function of the way a program expresses them. (War gaming and the "hangman" format are sample issues.) Also, the "humanizing" of the computer may serve for motivation or to reduce anxiety, but it also may become tedious, misleading, and counterproductive.

The summary of student performance can be dichotomous (win or lose), statistical (time expended or percent of items correct), or subjective (as in the evaluation of a simulation). It may be for student, teacher, or both.

SAMPLE SOFTWARE DOCUMENTATION SHEET

PROGRAM NAME: _____

SUBJECT AREA: _____

PROGRAM CLASS

____ isolated program

____ part of a ____ program cluster

INTENDED AUDIENCE(S)

____ teachers

____ students in grade(s) _____

____ other _____

_____ _____

PROGRAM USES: (1 = primary, 2 = secondary)

____ drill or practice

____ tutorial

____ simulation

____ instructional gaming

____ problem solving

____ informational

____ other _____

_____

CRITICAL PREREQUISITE SKILLS

_____

_____

NAME(S) OF PREREQUISITE PROGRAM(S)

_____

_____

_____

NAME(S) OF FOLLOW-UP PROGRAM(S)

_____

_____

_____

DATE OF THIS DOCUMENTATION

_____

PROGRAM SOURCE

name: _____

address: _____

_____

_____

phone: _____

program cost: _____

SYSTEM REQUIREMENTS

computer: _____

language: _____

memory needed: _____ bytes

input mode: cassette

____-inch diskette

cartridge

other _____

_____

output mode: color monitor

B/W monitor

printer

plotter

other _____

_____

CLASSROOM VALIDATION (describe)

_____

_____

_____

TIME FOR AVERAGE EXECUTION ____ minutes

OVER

SMC: 3/86

GENERAL DESCRIPTION OF THE PROGRAM (i.e., purpose of the program)

SPECIAL PROGRAM CHARACTERISTICS (e.g., use of graphics or sound, designed for use by groups of two or more persons at a time, level of difficulty can be modified by teacher (only) or student, exit from program is controlled by the teacher (only) or student, etc.)

DIRECTIONS FOR USE OF THE PROGRAM (e.g., directions not presented in the program itself but needed for program execution, directions regarding the use of the program within the total instructional process, notes regarding program options, etc.)

ANCILLARY MATERIALS REQUIRED (e.g., books, worksheets, charts, data lists, dice, geometric shapes, lab equipment, etc.)

ADDITIONAL DOCUMENTATION AVAILABLE (check box for each type)

- ☐ sample run (location _____)
- ☐ program listing (location _____)

APPENDIX C

MATHEMATICS TEXTBOOK EVALUATION

The procedure for textbook evaluation will vary depending on the size of the school and other factors unique to each school. In smaller schools, textbook selection may be the responsibility of one or two individuals, while in the larger schools a selection committee is usually formed. In either case the selection process should involve as many people as feasible.

The selection process usually becomes more detailed when committees are involved. However, there are certain general procedures usually followed regardless of the size of the selection committee. These might involve:

considering objectives,
selecting the evaluating committee,
developing evaluative criteria,
complying with legal provisions,
developing a rating form.

The evaluative criteria usually lead to the development of a rating form. The rating form often includes a rating scale which reflects the relative importance of each criterion in the rating process.

Criteria usually included on a rating form are:

content,
organization,
auxiliary materials,
physical characteristics,
special considerations.

In developing a rating form involving these criteria, a percent is attached to each criterion to establish its relative weight. For example, a rating of 1 to 5 could be applied to each criterion. By multiplying the relative weight (percent) by the rating (1 to 5) a weighted rating is established for each criterion. The weighted ratings of the criteria are totaled to produce an overall rating. This overall rating would then be a number from 1 to 5. A sample rating form follows.

TEXTBOOK EVALUATION FORM

Course Title _____

Textbook Title _____

Authors _____

Publisher _____

Copyright Date _____

Evaluator's Name _____

OVERALL RATING

CRITERION	Weight (%)	x	Rating	=	Weighted Rating
Content	_____	x	_____	=	_____
Organization	_____	x	_____	=	_____
Auxiliary Material	_____	x	_____	=	_____
Readability	_____	x	_____	=	_____
Physical Characteristics	_____	x	_____	=	_____
Other Considerations	_____	x	_____	=	_____
Must Total 100%	_____	x	_____	=	_____
	OVERALL RATING				_____

(Sum of the weighted rating will be a number from 1 to 5.)

A more comprehensive explanation of this procedure can be found in a handbook entitled The Evaluation of Mathematics Textbooks, which is available from: The National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

APPENDIX D

NCSM POSITION PAPER ON BASIC SKILLS

The currently popular slogan "Back to the Basics" has become a rallying cry of many who perceive a need for certain changes in education. The result is a trend that has gained considerable momentum and has initiated demands for programs and evaluations which emphasize narrowly defined skills.

Mathematics educators find themselves under considerable pressure from boards of education, legislatures, and citizens' groups who are demanding instructional programs which will guarantee acquisition of computational skills. Leaders in mathematics education have expressed a need for clarifying what are the basic skills needed by students who hope to participate successfully in adult society.

Any list of basic skills must include computation. However, the role of computational skills in mathematics must be seen in the light of the contributions they make to one's ability to use mathematics in everyday living. In isolation, computational skills contribute little to one's ability to participate in mainstream society. Combined effectively with the other skill areas, they provide the learner with the basic mathematical ability needed by adults.

The NCSM views basic mathematical skills as falling under ten vital areas. The ten skill areas are interrelated and many overlap with each other and with other disciplines. All are basic to pupils' development of the ability to reason effectively in varied situations.

This expanded list is presented with the conviction that mathematics education must not emphasize computational skills to the neglect of other critical areas of mathematics. The ten components of basic mathematical skills are listed below, but the order of their listing should not be interpreted as indicating either a priority of importance or a sequence for teaching and learning.

Furthermore, as society changes our ideas about which skills are basic also change. For example, today our students should learn to measure in both the customary and metric systems, but in the future the significance of the customary system will be mostly historical. There will also be increasing emphasis on when and how to use hand-held calculators and other electronic devices in mathematics.

Certainly drill and practice is a viable option for development of basic skills, but it is only one of many possible ways to bring about learning and to create interest and motivation in students. Learning centers, contracts, tutorial sessions, individual and small-group projects, games, simulations and community-based activities are some of the other options that can provide the opportunity to learn basic skills. Furthermore, to help students fully understand basic mathematical concepts, teachers should utilize the full range of activities and materials available, including objects the students can actually handle.

Because they do not supply sufficient information about how much mathematics a student knows, standardized tests are not the best instruments available for reporting individual pupil growth. Other alternatives such as criterion tests or competency tests must be considered. In criterion tests, items are generated which measure the specific objectives of the program and which establish the student's level of mastery of these objectives. Competency tests are designed to determine if the individual has mastered the skills necessary for a certain purpose such as entry into the job market. There is also need for open-ended assessments such as observations, interviews, and manipulative tasks to assess skills which paper and pencil tests do not measure adequately.

Large scale testing, whether involving all students or a random sample, can result in interpretations which have great influence on curriculum revisions and development. Test results can indicate, for example, that a particular mathematical topic is being taught at the wrong time in the student's development and that it might better be introduced later or earlier in the curriculum. The results might indicate that students are confused about some topic as a result of inappropriate teaching procedures. In any case, test results should be carefully examined by educators with special skills in the area of curriculum development.

The present paper represents a preliminary attempt by the National Council of Supervisors of Mathematics to clarify and communicate its position on basic mathematical skills. The NCSM position establishes a framework within which decisions on program planning and implementation can be made. It also sets forth the underlying rationale for identifying and developing basic skills and for evaluating pupils' acquisition of these competencies. The NCSM position underscores the fundamental belief of the National Council of Supervisors of Mathematics that any effective program of basic mathematical skills must be directed not "back" but forward to the essential needs of adults in the present and future.

Ten Basic Skill Areas

1. Problem solving
2. Applying mathematics to everyday situations
3. Alertness to the reasonableness of results
4. Estimation and approximation
5. Appropriate computational skills
6. Geometry
7. Measurement
8. Reading, interpreting, and constructing tables, charts, and graphs
9. Using mathematics to predict (probability and statistics)
10. Computer literacy

APPENDIX E

SUMMARY OF NCTM AN AGENDA FOR ACTION

Recommendations for
School Mathematics of the 1980s

Recommendation 1

PROBLEM SOLVING MUST BE THE FOCUS OF SCHOOL MATHEMATICS IN THE 1980s

Problem solving encompasses a multitude of routine and commonplace as well as nonroutine functions considered to be essential to the day-to-day living of every citizen. But it must also prepare individuals to deal with the special problems they will face in their individual careers.

Recommended Actions

1. The mathematics curriculum should be organized around problem solving.

2. Mathematics teachers should create classroom environments in which problem solving can flourish.

3. Appropriate curricular materials to teach problem solving should be developed for all grade levels.

4. Mathematics programs of the 1980s should involve students in problem solving by presenting applications at all grade levels.

Recommendation 2

THE CONCEPT OF BASIC SKILLS IN MATHEMATICS MUST ENCOMPASS MORE THAN COMPUTATIONAL FACILITY

There must be an acceptance of the full spectrum of basic skills and recognition that there is a wide variety of such skills beyond mere computational skills if we are to design a basic skills component of the curriculum that enhances rather than undermines education.

Common sense should dictate a reasonable balance among mental facilities with simple basic computations, paper-and-pencil algorithms for simple problems done easily and rapidly, and the use of calculators for more complex problems or those where problem analysis is the goal and cumbersome calculating is a limiting distraction.

*Copy of Agenda for Action available for $1.00 per copy from NCTM, 1906 Association Drive, Reston, VA 22091.

Professional knowledge of future trends; industrial, financial, engineering, and scientific needs, and the demands of daily life are all better arbiter of what is currently essential and what has become obsolete than our nostalgia as parents or teachers.

Recommended Actions

1. The full scope of what is basic should contain at least the ten basic skill areas identified by the National Council of Supervisors of Mathematics "Position Paper on Basic Skills." These areas are 1) problem solving; 2) applying mathematics in everyday situations; 3) alertness to the reasonableness of results; 4) estimation and approximation; 5) appropriate computational skills; 6) geometry; 7) measurement; 8) reading, interpreting, and constructing tables, charts, and graphs; 9) using mathematics to predict; and 10) computer literacy.

2. Teachers should incorporate estimation activities into all areas of the program on a regular and sustaining basis, in particular encouraging the use of estimating skills to pose and select alternatives and to assess what a reasonable answer may be.

3. Teachers should provide ample opportunities for students to learn communication skills in mathematics. They should systematically guide students to read mathematics and to talk about it with clarity.

4. The higher-order mental processes of logical reasoning, information processing, and decision making should be considered basic to the application of mathematics. Mathematics curricula and teachers should set as objectives the development of logical processes, concepts, and language.

Recommendation 3

MATHEMATICS PROGRAMS MUST TAKE FULL ADVANTAGE OF THE POWER OF CALCULATORS AND COMPUTERS AT ALL GRADE LEVELS

It is recognized that a significant portion of instruction in the early grades must be devoted to the direct acquisition of number concepts and skills without the use of calculators. However, when the burden of lengthy computations outweighs the educational contribution of the process, calculators should become readily available.

With the increasing availability of microcomputers at decreasing costs, it is imperative that schools play an active part in preparing students of the 1980s to live in a world in which more and more functions are being performed by computers.

Recommended Actions

1. All students should have access to calculators and increasingly to computers throughout their school mathematics programs.

2. The use of electronic tools such as calculators and computers should be integrated into the core mathematics curriculum.

3. Secondary school computer courses should be designed to provide the necessary background for advanced work in computer science.

4. School administrators and teachers should initiate interaction with the parents to achieve maximum benefit to the students through the coordinated home and school use of computers and calculators.

5. Educational users of electronic technology should demand a dual responsibility from manufacturers: the development of good software to promote the problem-solving abilities of the student and, eventually, the standardization and compatibility of hardware.

6. Teachers of other school subjects in which mathematics is applied should make appropriate use of calculators and computers in their instructional programs.

7. Certification standards should include preparation in computer literacy and the instructional uses of calculators and computers.

Recommendation 4

STRINGENT STANDARDS OF BOTH EFFECTIVENESS AND EFFICIENCY MUST BE APPLIED TO THE TEACHING OF MATHEMATICS

Modern technology and educational theory and research have made accessible to today's teachers approaches, materials and strategies that were not previously available. Teachers at all levels must learn to use these enriched varieties of instructional techniques, materials and resources to teach mathematics more effectively.

Recommended Actions

1. The major emphasis on problem solving in the curriculum must be accommodated by a rescheduling of the use of time in the classroom.

2. School administrators and parents must support the teacher's efforts to engage students more effectively in learning tasks.

3. Teachers should use diverse instructional strategies, materials and resources, i.e. small group work, video tapes, manipulatives.

Recommendation 5

THE SUCCESS OF MATHEMATICS PROGRAMS AND STUDENT LEARNING MUST BE EVALUATED BY A WIDER RANGE OF MEASURES THAN CONVENTIONAL TESTING

Test scores alone should not be considered synonymous with achievement or program quality. A serious danger to the education of our youth is the increasing tendency on the part of the public to assume that the sole objective of schooling is a high test score. This is often assumed without the critical knowledge of what is being tested or whether test items fit desired goals. The evaluation of problem-solving performance demands new approaches to measurement.

Recommended Actions

1. The evaluation of mathematics learning should include the full range of the program's goals, including skills, problem solving and problem-solving processes.

2. Parents should be regularly and adequately informed and involved in the evaluation process.

3. Teachers should become knowledgeable about, and proficient in, the use of a wide variety of evaluative techniques.

4. The evaluation of mathematics programs should be based on the program's goals, using evaluation strategies consistent with these goals.

5. The evaluation of materials for mathematics teaching should be an essential aspect of program planning.

6. Mathematics teachers must undergo continuing evaluation as a necessary component in improving mathematics programs.

Recommendation 6

MORE MATHEMATICS STUDY MUST BE REQUIRED FOR ALL STUDENTS AND A FLEXIBLE CURRICULUM WITH A GREATER RANGE OF OPTIONS SHOULD BE DESIGNED TO ACCOMMODATE THE DIVERSE NEEDS OF THE STUDENT POPULATION

Technical and vocational training at different levels assumes more and diverse mathematical backgrounds. For those persons whose formal education will end with high school, the needs of the citizen and the consumer for increasing mathematical sophistication dictate a collection of courses based on consumer and career needs, computer literacy, and quantitative literacy.

It is important that recommended programs permit lateral movement and not strictly "track" students, trapping them in a linear pattern that does not permit change to another path. Flexibility is vital, and the key is to keep options open as long as possible.

It is naive to suppose that just providing mathematics courses as electives will serve for equality of opportunity. All reasonable means should be employed to assure that everyone will have the foundations of mathematical learning essential to fulfilling his/her potential as a productive citizen. The currently underrepresented groups should be especially encouraged and helped.

The student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world.

Recommended Actions

1. Schools should increase the amount of time students spend in the study of mathematics.

2. In secondary schools, the curriculum should become more flexible, permitting a greater number of options for a diversified student population.

3. Mathematics teachers and college mathematicians should reevaluate the role of calculus in the differentiated mathematics programs.

4. The curriculum that stresses problem solving must pay special heed to the developmental sequence best suited to achieving process goals, not just content goals.

Recommendation 7

MATHEMATICS TEACHERS MUST DEMAND OF THEMSELVES AND THEIR COLLEAGUES A HIGH LEVEL OF PROFESSIONALISM

Educators can gain the support of society and the rewards of a truly effective performance by developing, defining, and enforcing professional standards in terms of highly competent professional performance rather than by any other norm. This must be done to provide the nation, its young people, and its future with the mathematics programs worthy of them and of that future.

Within mathematics teaching--

-- there are already many well-prepared and effective teachers who provide outstanding professional leadership;

-- there are also many teachers who are motivated and desire to improve but who lack adequate preparation and must be given the necessary support to become fully qualified and to improve;

-- there are, however, some teachers whose attitudes and functioning are less than a professional level. In the best interests of students and society, the number of such teachers must be reduced immediately.

Even the best prepared, competent, and dedicated teachers must continue their development to keep abreast of changing needs, tools, and conditions. School administrators have the responsibility to make this possible by providing continuing inservice education and by encouraging teachers to take full advantage of opportunities for maintaining their competence that are offered by professional organizations and universities.

Recommended Actions

1. Every mathematics teacher should accept responsibility for maintaining teaching competence.

2. Certification standards for mathematics teaching should be revised and upgraded to incorporate the needs reflected in these recommendations.

Recommendation 8

PUBLIC SUPPORT FOR MATHEMATICS INSTRUCTION MUST BE RAISED TO A LEVEL COMMENSURATE WITH THE IMPORTANCE OF MATHEMATICAL UNDERSTANDING TO INDIVIDUALS AND SOCIETY

The rededication of teachers called for in Recommendation 7 will be meaningless or impossible if society is not committed to supporting the professionalism it rightfully demands. Today, teachers' financial incentives are neither commensurate with the responsibilities they carry nor adequate to attract and retain them in a demanding, crucial, and sometimes burdensome profession. Few school systems have truly adequate supervisory and material support for the teacher in the maintenance and improvement of the instructional environment. More and more, teachers feel the lack of parental understanding of the complexity of their task and the lack of parental cooperation and support in their efforts to instruct children. Furthermore, governmental support for improving the quality of mathematics teaching has dwindled.

At present, there are too many unnecessary obstacles to the effective functioning of teacher and student in a true teaching/learning interaction. These include more and more time required for unproductive recordkeeping; many unmotivated, undisciplined students; a lack of parental support; ambivalence and vacillation in government regulations; shifting societal priorities; the lack of home and school agreement on out-of-school assignments.

There are also obstacles faced by school administrators and by the policy-making bodies, the school boards, whose ultimate responsibility is the effectiveness of the school district. It is recognized that in many instances confusions of legalism and court decisions impose constraints on the ability of these people to accomplish what they agree should be done. Society today is troubled with many complexities. All institutions are threatened. These obstacles should not be an excuse for inaction; rather, in the light of pressing student needs it is imperative that they be surmounted.

Recommended Actions

1. Society must provide the incentives that will attract and retain competent, fully prepared, qualified mathematics teachers.

2. Parents, teachers, and school administrators must establish new and higher standards of cooperation and teamwork toward the common goal of educating each student to his/her highest potential.

3. Government at all levels should operate to facilitate, not dictate, the attainment of goals agreed on cooperatively by the public's representatives and the professionals.

APPENDIX F

MINIMUM SKILLS FOR COLLEGE ALGEBRA

The following section contains material developed through an Academic Skills Project involving several high school teachers in North Dakota and Minnesota. This material is designed to determine whether a student is adequately prepared to take college algebra. These materials will help the teacher provide specific objectives that a student should possess before enrolling in college algebra.

The test that follows should be used to evaluate the students achievement of these objectives. The test is designed to be given to students at the end of their sophomore year. The average student will score between 13 and 15 out of 36 possible points. A score of 28 is adequate to anticipate success in college algebra. A student completing an Algebra II course should score 28 or more on the test.

For more information write:

Dr. Glenn Prigge
Mathematics Department
University of North Dakota
University Station, P.O. Box 8162
Grand Forks, ND 58202
Phone: (701) 777-4612

Members of the Mathematics Skill Task Force

Dr. Glenn Prigge
Grand Forks, ND

Gary Bethke
Grand Forks, ND

Jeff Hoplin
Grand Forks, ND

Jim Sheridan
Crookston, MN

Mike Kern
Grafton, ND

Duane Hanson
East Grand Forks, MN

Milo Benson
Fargo, ND

Ed Halvorson
Devils Lake, ND

Jane Reff
Fargo, ND

Bruce Moen
Hillsboro, ND

Luann Stehr
Minot, ND

INFORMATION TO BE GIVEN TO PEOPLE WHO WILL ADMINISTER THE TEST

1. The purpose of this test is to give students an idea whether or not they presently possess the math skills required to enroll in college algebra and be successful.

2. It is recommended that the test be given to 10th, 11th, and 12th grade students presently enrolled in an Algebra I or higher course. If your school is small enough, you may give the test to all the sophomores as well as those 11th and 12th graders enrolled in Algebra I or above. This would be ideal.

3. If you are interested in using the test with 9th graders, please use it at the end of the school year and remind the students of the purpose of the test.

4. The test should be given to the students before they register for their next year's courses. One of the purposes of this test is to counsel students to take more math. Therefore, it is imperative that students see their results at least one to two weeks before registering for the following year's courses. Schools may use their own judgment in planning the most appropriate test date.

5. The suggested time limit is 45 minutes of writing time. Also realize a few minutes are needed for administrative procedures. Each student should have three sheets of scratch paper and a pencil. Calculators are permissible but are not required because they probably won't help.

6. Soon after the test is given and corrected, the results should be presented to the students.

PREREQUISITE SKILLS NECESSARY FOR COLLEGE ALGEBRA

I. The Real Number System

1. Numbers

a. Identify various kinds of real numbers.

b. Convert from decimal to fractional or percent notation, and vice versa.

c. Determine which of two numbers is greater and indicate which, using $<$ or $>$.

2. Four Basic Operations

a. Add, subtract, multiply, and divide real numbers.

b. Find additive and multiplicative inverses of real numbers.

c. Recognize impossible divisions (divisions by 0).

d. Apply fundamental order of operations.

3. The Distributive Laws

a. Evaluate an expression like ab + cd when numbers are specified for the letters.

b. Use the distributive laws to multiply expressions like 5 and (x + 9).

c. Use the distributive laws to factor expressions like 4x - 12.

4. Multiplying by -1 and Simplifying

a. Identify the terms of an expression.

b. Simplify expressions by removing parentheses and collecting like terms.

c. Simplify expressions containing parentheses within parentheses.

5. Exponential Notation

a. Rewrite expressions with or without whole-number exponents.

b. Rewrite expressions with or without negative integers as exponents.

6. Properties of Exponents

   a. Use exponential notation in multiplication and division.

   b. Use exponential notation in raising a power to a power.

II. Linear Equations

7. Solving Equations

   a. Tell the difference between an expression and an equation.

   b. Recognize what is meant by a solution or root of an equation.

   c. Solve equations using the addition and multiplication principles.

   d. Solve certain equations containing parentheses.

8. Graphs

   a. Plot points on a plane, given their coordinates.

   b. Determine whether an ordered pair of numbers is a solution of an equation with two variables.

   c. Given an equation $y = mx + b$, determine the slope and y-intercept and graph the equation.

   d. Graph linear equations with a missing variable (parallel to an axis).

   e. Determine which equations are linear.

   f. Graph linear equations using intercepts.

   g. Given two points of a line, find its slope.

   h. Identify lines with zero slope and lines that have no slope.

9. Determining Equations of Lines

   a. Given the slope of a line and a point on the line, find an equation of the line.

   b. Given two points of a line, find an equation of the line.

   d. Given a line and a point, find an equation of the line parallel to the given line and containing the point, and an equation of the line perpendicular to the given line and containing the point.

III. Systems of Equations

10. Systems of Equations in Two Variables

a. Determine whether an ordered pair is a solution of a system of two equations.

b. Solve systems of two linear equations graphically.

c. Solve systems of equations in two variables by the substitution method.

d. Solve systems of equations in two variables by the addition method.

11. Systems of Equations in Three Variables

a. Determine whether an ordered triple is a solution of a system of three equations in three variables.

b. Solve systems of three equations in three variables.

12. Inconsistent and Dependent Equations

a. Determine whether a system of two or three linear equations is consistent or inconsistent.

b. Determine whether a system of two or three linear equations is dependent or independent.

IV. Polynomials and Functions

13. Some Properties of Polynomials

a. Identify the terms and coefficients of a polynomial.

b. Determine the degree of each term of a polynomial and the degree of the polynomial.

c. Arrange a polynomial in ascending or descending powers of a given variable.

d. Identify monomials, binomials, and trinomials.

14. Calculations With Polynomials

a. Add and subtract polynomials.

b. Rename the additive inverse of a polynomial without using parentheses.

15. Multiplying Polynomials

    a. Multiply monomials.

    b. Multiply a binomial by a monomial.

    c. Multiply any two polynomials.

16. Special Products of Polynomials

    a. Multiply two binomials mentally.

    b. Square a binomial mentally.

    c. Multiply, mentally, the product of the sum and the difference of the same expressions, $(x + y)(x - y)$.

17. Factoring

    a. Factor polynomials whose terms have a common factor.

    b. Factor polynomials with four terms by grouping.

    c. Factor differences of two squares.

    d. Factor trinomial squares.

    e. Factor trinomials of the type $x^2 + ax + b$.

    f. Factor trinomials of the type $ax^2 + bx + c$.

    g. Factor differences of squares having more than two terms.

    h. Factor sums and differences of two cubes.

18. Solving Equations by Factoring

    a. Solve equations by factoring and using the principles of zero products.

19. Functions

    a. Given a mapping, determine whether it is a function.

    b. Given a function defined by a formula, find function values (range) for specified domain values.

    c. Recognize the graph of a function.

V. Fractional Expressions and Equations

20. Multiplying and Dividing

    a. Multiply, divide, and simplify fractional expressions.

21. Least Common Multiples

    a. Find the LCM of several numbers by factoring.

    b. Add numbers (fractional notation) by first finding a common denominator.

    c. Find the LCM of several algebraic expressions by factoring.

22. Addition and Subtraction

    a. Add and subtract fractional expressions having the same denominator.

    b. Add and subtract fractional expressions whose denominators are additive inverses of each other.

    c. Add and subtract fractional expressions having different denominators.

    d. Simplify combined additions and subtractions of fractional expressions.

23. Complex Fractional Expressions

    a. Simplify complex fractional expressions.

24. Division of Polynomials

    a. Divide a polynomial by a monomial.

    b. Divide a polynomial by a divisor that is not a monomial, and if there is a remainder, express the result in two ways.

25. Solving Fractional Equations

    a. Solve fractional equations.

    b. Check for extraneous roots.

26. Variation

    a. Given a description of direct variation, find an equation of variation.

    b. Solve problems involving direct variation.

    c. Given a description of inverse variation, write an equation of variation.

    d. Solve problems involving inverse variation.

VI. Exponents, Powers, and Roots

27. Scientific Notation

a. Simplify expressions with integer exponents.

b. Convert between scientific notation and ordinary decimal notation.

c. Multiply and divide using scientific notation.

d. Use scientific notation in estimating.

28. Radical Expressions

a. Find principal square roots and their additive inverses.

b. Simplify radical expressions with perfect-square radicands.

c. Find cube roots.

d. Simplify expressions involving odd and even roots.

29. Multiplying and Simplifying With Radicals

a. Simplify radical expressions by factoring, assuming that all expressions represent nonnegative numbers.

b. Multiply and simplify radical expressions.

c. Approximate irrational numbers.

30. Dividing and Simplifying With Radicals

a. Simplify radical expressions having a fraction for a radicand.

b. Divide and simplify expressions.

31. Addition and Subtraction Involving Radicals

a. Add or subtract radicals and simplify.

32. Rationalizing Denominators

33. Rational Numbers as Exponents

a. Write expressions with or without fractional exponents.

b. Write expressions without negative exponents.

c. Use the laws of exponents with fractional exponents.

d. Use fractional exponents to simplify radical expressions.

34. Radical Equations

a. Solve radical equations and check extraneous roots.

35. The Complex Numbers

a. Express imaginary numbers as bi, where b is a real number, and complex numbers as a + bi, where a and b are real numbers.

b. Add, subtract, multiply, and divide complex numbers.

c. Find conjugates of complex numbers.

d. Determine whether a given complex number is a solution of an equation.

VII. Quadratic Equations and Functions

36. Quadratic Equations

a. Solve equations of the type $ax^2 = k$.

b. Solve equations of the type $ax^2 + bx = 0$.

c. Solve equations of the type $ax^2 + bx + c = 0$.

d. Solve quadratic equations using the quadratic formula.

37. Quadratic Functions and Their Graphs

a. Graph functions of the form $y = ax^2$ or $f(x) = ax^2$.

b. Graph functions like $f(x) = a(x - h)^2$.

c. Graph functions like $f(x) = a(x - h)^2 + k$ and determine the vertex, line of symmetry, and the maximum or minimum function values.

d. Given the quadratic function $f(x) = ax^2 + bx + c$, complete the square to put it into the form $f(x) = a(x - h)^2 + k$, and then graph the function.

e. Find the x-intercepts of a quadratic function.

38. The Distance Formula

a. Given the lengths of two sides of a right triangle, give the length of the third side.

b. Use the distance formula to find the distance between two points.

c. Find the midpoint of a line segment.

VIII. Inequalities and Absolute Value

39. Solving Inequalities

a. Determine whether a number is a solution of an inequality.

b. Solve simple inequalities in one variable, using the addition principle, and graph them on a number line.

c. Solve inequalities using the multiplication principle.

40. Absolute Value

a. Find the distance between numbers using absolute value.

b. Solve equations and inequalities involving absolute value.

IX. Problem Solving and Formulas

41. Translating Statements

a. Translate statements to linear equations.

b. Translate statements to system of equations (two or three variables) and solve the system.

c. Translate statements into quadratic equations and solve the equation.

d. Solve maximum and minimum problems.

42. Formulas

a. Solve formulas for specified variables.

b. Evaluate formulas for given values.

COLLEGE ALGEBRA MINIMUM SKILLS TEST INSTRUCTIONS

The purpose of this test is to give students an idea whether or not they presently possess the mathematics skills required to enroll in college algebra and be successful. You should do the best you possibly can on the test in order to reflect your true algebra skill.

If your results are lower than desirable, you need not be discouraged. Even though this shows you need to improve your algebra skills, you still have time to take advantage of the mathematics courses your school offers you in order for you to obtain these necessary skills.

If you do well on the test, this indicates your skill level is adequate for being successful in college algebra. However, these results will not indicate any form of success in calculus.

Materials: You should have a test booklet, response sheet, scratch paper, and pencil. Optional: eraser, calculator.

Directions:
1. Select the best answer for each item
2. Record on the response sheet
3. Respond to all items - there is no penalty for guessing

COLLEGE ALGEBRA MINIMUM SKILLS TEST

1. Find the decimal notation for 12.5%.

   a. 12.5 b. 1.25% c. .125

   d. 1,250 e. none of these

2. Add: $\frac{-5}{2} + \frac{-7}{4}$

   a. $\frac{-12}{8}$ b. $\frac{-17}{4}$ c. $\frac{-12}{4}$

   d. $\frac{17}{4}$ e. none of these

3. Multiply: (-4.1) (8.2)

   a. -33.62 b. 33.62 c. -32.02

   d. -32.2 e. none of these

4. Divide and simplify: $\frac{y^3}{y^{-4}}$

   a. $\frac{1}{y^{+7}}$ b. y^{-7} c. y^{-1}

   d. y^7 e. none of these

5. Solve: $y + \frac{1}{4} = \frac{-3}{4}$

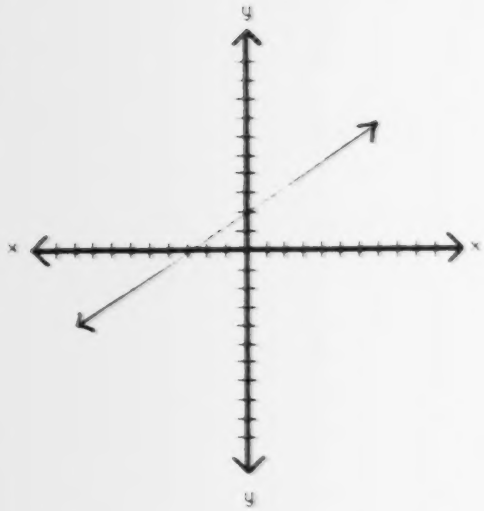
   a. 1 b. -1 c. $\frac{1}{2}$

   d. $\frac{-1}{2}$ e. none of these

6. Solve: $8x - (3x - 5) = 40$

   a. 9 b. 3 c. 7

   d. 5 e. none of these

7. 

This graph represents the equation:

a. $y = \frac{2}{3}x + 2$

b. $y = \frac{3}{2}x + 2$

c. $y = 3x + 2$

d. $y = 2x + 3$

e. none of these

8. The slope of the equation $y = \frac{3}{4}x + \frac{17}{4}$ is:

a. $\frac{17}{4}$ b. $\frac{4}{3}$ c. $\frac{-3}{4}$

d. 3 e. none of these

9. Solve: $2x + y = 1$
$x - y = -7$

a. $x = 1, y = -1$ b. $x = -2, y = 5$ c. $x = 0, y = 1$

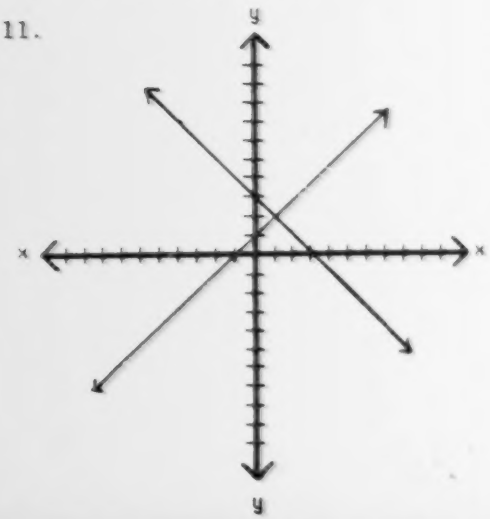
d. $x = 2, y = -3$ e. none of these

10. Solve: $x + y + z = 2$
$x - 2y - z = 2$
$3x + 2y + z = 2$

a. $x = 1, y = -2, z = 3$ b. $x = 1, y = 1, z = 0$

c. $x = 1, y = -2, z = 0$ d. $x = 0, y = -4, z = 6$

e. none of these

11. 

This graph represents the solution for which system of equations?

a. $y = 5$
$x + y = 3$

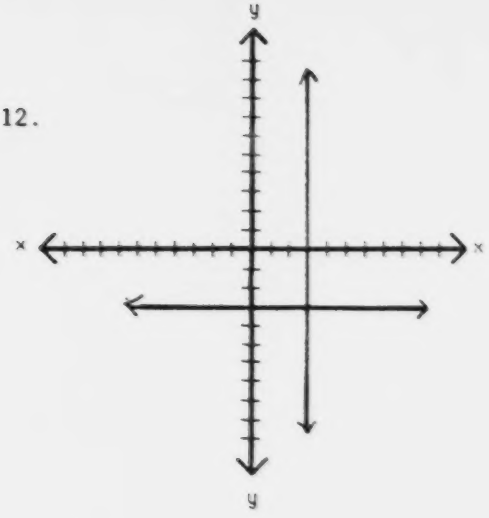
b. $y = x$
$x - y = 1$

c. $y - x = 1$
$2y + x = 3$

d. $y - x = 1$
$x + y = 3$

e. none of these

12.



The graph represents the solution for which system of equations?

a. $y = 3$, $x = -3$ b. $y = -3$, $x = -3$

c. $y = +3$, $x = +3$ d. $y = 0$, $x = 0$

e. none of these

13. Multiply: $(5x + 7)(3x - 6)$

a. $8x^2 + 9x - 42$ b. $8x^2 - 9x - 42$ c. $15x^2 + 11x - 42$

d. $15x^2 - 9x - 42$ e. none of these

14. Factor completely: $m^2 - 64$

a. $(m - 8)^2$ b. $(m + 8)(m - 8)$ c. $(m + 8)^2$

d. $(m + 4)(m - 16)$ e. none of these

15. The solution set for $2x^2 = 3 - 5x$ is:

a. $\left\{\frac{-3}{2}, 1\right\}$ b. $\left\{\frac{2}{3}, 1\right\}$ c. $\left\{\frac{1}{2}, -3\right\}$

d. $\{2, -3\}$ e. none of these

16. If $f(x) = 3x^3 - 7x + 2$, then $f(2)$ is:

a. 12 b. 6 c. 69

d. 15 e. none of these

17. Divide and simplify: $\dfrac{y^2 - 16}{2y + 8} \div \dfrac{y + 4}{y - 4}$

a. $\dfrac{y + 4}{2}$ b. $\dfrac{2y + 8}{y^2 - 8y + 16}$ c. $\dfrac{y^2 - 8y + 16}{2y + 8}$

d. $\dfrac{1}{2}$ e. none of these

18. Simplify: $\dfrac{1 - \frac{1}{x^2}}{1 - \frac{1}{x}}$

a. $\dfrac{x}{x + 1}$ b. $\dfrac{x - 1}{x}$ c. $\dfrac{1}{x}$

d. $\dfrac{x + 1}{x}$ e. none of these

19. Solve: $\dfrac{2}{x - 1} = \dfrac{3}{x + 3}$

a. 9 b. $\emptyset$ c. 3

d. -3 e. none of these

20. The time required to drive a certain distance varies inversely as the speed. If you can make a trip in 4 hours at 45 mph, how long will it take you at 60 mph?

a. $4\frac{2}{3}$ hours b. $2\frac{5}{8}$ hours c. $2\frac{4}{5}$ hours

d. 3 hours e. none of these

21. Add: $(9\sqrt{5} - 7\sqrt{3}) + (5\sqrt{5} + 7\sqrt{3})$

a. $\sqrt{6} + 4\sqrt{5} + 8$ b. $\sqrt{7} \cdot 30 - 9\sqrt{30} - 149$

c. $\sqrt{6} - 14\sqrt{5} + 8$ d. $14\sqrt{5}$

e. none of these

22. Multiply and simplify: $\sqrt[3]{-x^4}\ \sqrt[3]{8x^5}$

a. $2x^3$ b. $\sqrt[6]{-8x^9}$ c. $-2x^3$

d. $\sqrt[3]{8x^9}$ e. none of these

23. Rationalize the denominator and simplify: $\dfrac{6}{1 + \sqrt{3}}$

a. $-3 + 3\sqrt{3}$ b. $-3 - 3\sqrt{3}$ c. $\dfrac{6 - \sqrt{3}}{-2}$

d. $\dfrac{3 - 3\sqrt{3}}{4}$ e. none of these

24. Multiply and simplify the following complex numbers: $(3 - 2i)(4 + 3i)$

a. $12 + i + 6i^2$ b. $18 + i$ c. $12 - 6i$

d. $12 - 6i^2$ e. none of these

25. Solve by the Quadratic formula: $3x^2 + 5x - 11 = 0$

a. $\frac{-5 \pm \sqrt{157}}{2}$ b. $\frac{5 \pm \sqrt{157}}{6}$ c. ± 5

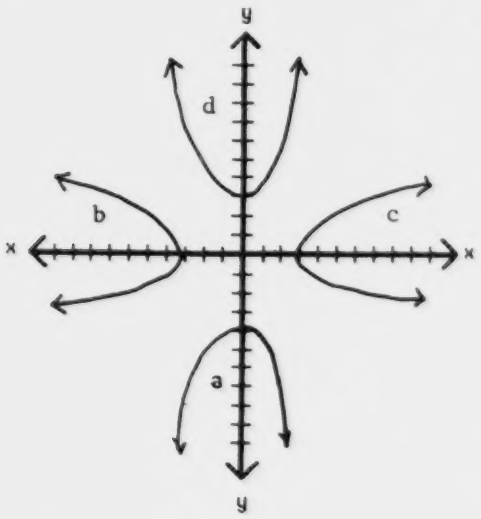
d. $\frac{-5 \pm \sqrt{157}}{6}$ e. none of these

26. Given $f(x) = -3(x + 4)^2 - 4$, find the coordinates of the vertex.

a. vertex (-4,-4) b. vertex (-4,4) c. vertex (4,-4)

d. vertex (-3,-4) e. none of these

27. The graph of the equation $y = x^2 + 3$ looks like:



e. none of these

28. Given the points A (7,-2) and B (1,6), find the distance from A to B.

a. 100 b. 7 c. 10

d. 25 e. none of these

29. Determine the solution to the following inequality: $3 - x < 2$

a. $x > 1$ b. $x > 5$ c. $x < 5$

d. $x < 1$ e. none of these

30. Determine the solution to the following inequality: $5x \leq \frac{3}{2}$

a. $x \leq \frac{15}{2}$ b. $x \leq \frac{10}{3}$ c. $x \geq \frac{15}{3}$

d. $x \leq \frac{3}{10}$ e. none of these

31. Solve: $|x + 2| = 4$

a. 2 b. -6 c. 2 and -6

d. 2 and -8 e. none of these

32. If x is an integer for which $x + 2 > -5$, then

a. x is the set $\{-1, -2, -3, -4, -5, \ldots\}$

b. x is the set $\{-7, -8, -9, -10, \ldots\}$

c. x is the set $\{-6, -5, -4, -3, -2, \ldots\}$

d. x is the set $\{-2, -1, 0, +1, +2, \ldots\}$

e. none of these

33. The square of a number plus 60 is sixteen times the number. Find the number(s).

a. -6 and -10 b. 6 and 10 c. 6 and -10

d. -6 and 10 e. none of these

34. Grafton, North Dakota claims to have the purest drinking water in the world. It is 99.9% pure. If you have 240 liters of water from Grafton, how many liters of pure water do you have?

a. 50 liters b. 245 liters c. 239 liters

d. 239.76 liters e. none of these

35. Solve for e : $F = \frac{e}{4}(E + g)$

a. $e = \frac{4F - eg}{e}$ b. $e = 4F - (E + g)$ c. $e = \frac{4F}{Eg}$

d. $e = \frac{4F}{E + g}$ e. none of these

36. Given: $A = \pi r^2 + 2\pi rh$, $r = 2$ and $A = 64\pi$, find h.

a. 8 b. 30π c. 15

d. 16 e. none of these

ANSWER SHEET FOR COLLEGE ALGEBRA MINIMUM SKILLS TEST

DATE_____ TEST SCORE_____

NAME (PRINT)_____

NAME OF SCHOOL AND CITY_____
(school) (city)

Class level (circle one) FR. SOPH. JR. SR.

Sex (circle one) MALE FEMALE

Darken the circle that corresponds to the correct answer.

	a	b	c	d	e		a	b	c	d	e		a	b	c	d	e
1.	0	0	0	0	0	13.	0	0	0	0	0	25.	0	0	0	0	0
2.	0	0	0	0	0	14.	0	0	0	0	0	26.	0	0	0	0	0
3.	0	0	0	0	0	15.	0	0	0	0	0	27.	0	0	0	0	0
4.	0	0	0	0	0	16.	0	0	0	0	0	28.	0	0	0	0	0
5.	0	0	0	0	0	17.	0	0	0	0	0	29.	0	0	0	0	0
6.	0	0	0	0	0	18.	0	0	0	0	0	30.	0	0	0	0	0
7.	0	0	0	0	0	19.	0	0	0	0	0	31.	0	0	0	0	0
8.	0	0	0	0	0	20.	0	0	0	0	0	32.	0	0	0	0	0
9.	0	0	0	0	0	21.	0	0	0	0	0	33.	0	0	0	0	0
10.	0	0	0	0	0	22.	0	0	0	0	0	34.	0	0	0	0	0
11.	0	0	0	0	0	23.	0	0	0	0	0	35.	0	0	0	0	0
12.	0	0	0	0	0	24.	0	0	0	0	0	36.	0	0	0	0	0

KEY FOR COLLEGE ALGEBRA MINIMUM SKILLS TEST

DATE_____ TEST SCORE_____

NAME (PRINT) Key

NAME OF SCHOOL AND CITY_____
(school) (city)

Class level (circle one) FR. SOPH. JR. SR.

Sex (circle one) MALE FEMALE

Darken the circle that corresponds to the correct answer.

	a	b	c	d	e		a	b	c	d	e		a	b	c	d	e
1.	0	0	●	0	0	13.	0	0	0	●	0	25.	0	0	0	●	0
2.	0	●	0	0	0	14.	0	●	0	0	0	26.	●	0	0	0	0
3.	●	0	0	0	0	15.	0	0	●	0	0	27.	0	0	0	●	0
4.	0	0	0	●	0	16.	●	0	0	0	0	28.	0	0	●	0	0
5.	0	●	0	0	0	17.	0	0	●	0	0	29.	●	0	0	0	0
6.	0	0	●	0	0	18.	0	0	0	●	0	30.	0	0	0	●	0
7.	●	0	0	0	0	19.	●	0	0	0	0	31.	0	0	●	0	0
8.	0	0	0	0	●	20.	0	0	0	●	0	32.	0	0	●	0	0
9.	0	●	0	0	0	21.	0	0	0	●	0	33.	0	●	0	0	0
10.	●	0	0	0	0	22.	0	0	●	0	0	34.	0	0	0	●	0
11.	0	0	0	●	0	23.	●	0	0	0	0	35.	0	0	0	●	0
12.	0	0	0	0	●	24.	0	●	0	0	0	36.	0	0	●	0	0

APPENDIX G

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